Vectors

EXERCISES

ELEMENTRY

Q.1 (4)

$$\overrightarrow{AB} = (6-2)\hat{\boldsymbol{i}} + (-3+9)\hat{\boldsymbol{j}} + (8+4)\hat{\boldsymbol{k}} = 4\boldsymbol{i} + 6\boldsymbol{j} + 12\boldsymbol{k}$$

$$|\overrightarrow{AB}| = \sqrt{16+36+144} = 14.$$

Q.2 (1)

$$\begin{vmatrix} 1 & 2 & 3 \\ \lambda & 4 & 7 \\ -3 & -2 & -5 \end{vmatrix} = 0 \Rightarrow \lambda = 3.$$

Q.3 (1)

$$a = 4\hat{i} + 2\hat{j} - 4\hat{k} \Rightarrow |a| = \sqrt{16+16+4} = 6$$

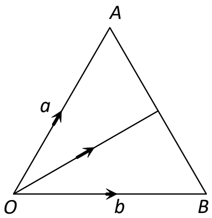
$$b = -3\hat{i} + 2\hat{j} + 12\hat{k} \Rightarrow |b| = \sqrt{144+4+9} = \sqrt{157}$$

$$c = -\hat{i} - 4\hat{j} - 8\hat{k} \Rightarrow |c| = \sqrt{64+16+1} = 9$$

Hence perimeter is $15 + \sqrt{157}$.

Q.4 (4)

Since given that $\overrightarrow{AC} = 3\overrightarrow{AB}$. It means that point divides externally. Thus $\overrightarrow{AC} : \overrightarrow{BC} = 3:2$



Hence $\overrightarrow{OC} = \dfrac{3.b - 2.a}{3-2} = 3b - 2a.$

Q.5 (2)

Let position vector of D is $xi + yj + zk$, then $\overrightarrow{AB} = \overrightarrow{DC}$

$$\Rightarrow -2\hat{j} - 4\hat{k} = (7-x)\hat{i} + (7-y)\hat{j} + (7-z)\hat{k}$$

Hence position vector of will be .

$$\Rightarrow x = 7,\ y = 9,\ z = 11.$$

Q.6 (3)

If x be the position vector of B, then a divides AB in the ratio 2 : 3.

$$\boldsymbol{a} = \frac{2\boldsymbol{x} + 3(\boldsymbol{a} + 2\boldsymbol{b})}{2+3}$$

$$5\boldsymbol{a} - 3\boldsymbol{a} - 6\boldsymbol{b} = 2\boldsymbol{x} \Rightarrow \boldsymbol{x} = \boldsymbol{a} - 3\boldsymbol{b}.$$

Q.7 (4)

$\overrightarrow{AB} = -2\hat{j}$, Here $\overrightarrow{BC} = (a-1)\hat{i} + (b+1)\hat{j} + c\hat{k}$

The points are collinear, then $\overrightarrow{AB} = k(\overrightarrow{BC})$

$$-2\hat{j} = \hat{k}\{(a-1)\hat{i} + (b+1)\hat{j} + c\hat{k}\}$$

On comparing, $k(a-1) = 0$, $k(b+1) = -2$, $kc = 0$.

Hence $c = 0$, $a = 1$ and b is arbitrary scalar.

Q.8 (1)

Let $a = x\hat{i} + y\hat{j} + z\hat{k}$.

Then $(a.\hat{i})\hat{i} + (a.\hat{j})\hat{j} + (a.\hat{k})\hat{k} = a.$

Q.9 (4)

Let $r = x\hat{i} + y\hat{j} + z\hat{k}.$

Since $r.\hat{i} = r.\hat{j} = r.\hat{k} \Rightarrow x = y = z$ .....(i)

Also $|r| = \sqrt{x^2 + y^2 + z^2} = 3 \Rightarrow x = \pm\sqrt{3}$, {By (i)}

Hence the required vector $r = \pm\sqrt{3}(\hat{i} + \hat{j} + \hat{k}).$

Trick : As the vector $\pm\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$ satisfies both the conditions.

Q.10 (4)

Parallel vector $= (2+b)\hat{i} + 6\hat{j} - 2\hat{k}$

Unit vector $= \dfrac{(2+b)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{b^2 + 4b + 44}}$

According to the condition, $1 = \dfrac{(2+b) + 6 - 2}{\sqrt{b^2 + 4b + 44}}$

$$\Rightarrow b^2 + 4b + 44 = b^2 + 12b + 36 \Rightarrow 8b = 8 \Rightarrow b = 1.$$

Q.11 (1)

$$(a+b+c)^2 = 0$$

$$\Rightarrow |a|^2 + |b|^2 + |c|^2 + 2a.b + 2b.c + 2c.a = 0$$

$$\Rightarrow 9 + 1 + 16 + 2(a.b + b.c + c.a) = 0$$

$$\Rightarrow a.b + b.c + c.a = -\frac{26}{2} = -13.$$

Q.12 (2)

$$\cos\theta = \frac{3(2) + (1)(-2) + 2(4)}{\sqrt{9+1+4}\sqrt{4+4+16}} = \frac{12}{\sqrt{14}\sqrt{24}} = \frac{6}{\sqrt{14}\sqrt{6}}$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{\sqrt{7}} \Rightarrow \sin\theta = \frac{2}{\sqrt{7}} \Rightarrow \theta = \sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$$

Q.13 (2)

$$(a+2b).(5a-4b)=0$$

or $5a^2+6a.b-8b^2=0$

or $6a.b=3$, $(\because a^2=1, b^2=1)$

$\therefore a.b=\frac{1}{2}$ or $|a||b|\cos\theta=\frac{1}{2}$

$\therefore \cos\theta=\frac{1}{2}$, $\quad \therefore \theta=60^o$.

Q.14 (3)

$a.b=(2-4-\lambda)=0 \Rightarrow \lambda=-2.$

Q.15 (1)

Projection of $x\hat{i}-\hat{j}+\hat{k}$ on $2\hat{i}-\hat{j}+5\hat{k}$

$$=\frac{(x\hat{i}-\hat{j}+\hat{k})(2\hat{i}-\hat{j}+5\hat{k})}{\sqrt{4+1+25}}=\frac{2x+1+5}{\sqrt{30}}$$

But, given $\frac{2x+6}{\sqrt{30}}=\frac{1}{\sqrt{30}}$

$\Rightarrow 2x+6=1 \Rightarrow x=\frac{-5}{2}$

Q.16 (1)

$a=\hat{i}+\hat{j}-3\hat{k}$, $b=-2\hat{i}+2\hat{j}+2\hat{k}$

$$a\times b=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -3\\ -2 & 2 & 2\end{vmatrix}=8\hat{i}+4\hat{j}+4\hat{k}$$

Hence unit vector $=\pm\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$

Q.17 (2)

$$|a\times\hat{i}|^2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ 1 & 0 & 0\end{vmatrix}^2, \text{(Since } a=a_1\hat{i}+a_2\hat{j}+a_3\hat{k})$$

$=|a_3\hat{j}-a_2\hat{k}|^2=a_3^2+a_2^2$

Similarly, $|a\times\hat{j}|^2=a_1^2+a_3^2$ and $|a\times\hat{k}|^2=a_1^2+a_2^2$

Hence the required result can be given as

$2(a_1^2+a_2^2+a_3^2)=2|a|^2$.

Q.18 (3)

We know that $(a\times b)^2+(a.b)^2=|a|^2|b|^2$

$\therefore 144=16|b|^2 \Rightarrow |b|=3.$

Q.19 (2)

$(a\times b)^2=(|a||b|\sin\theta)^2$

$$=(4.2\sin 30^o)^2=\left(8.\frac{1}{2}\right)^2=4^2=16$$

Q.20 (2)

If angle between b and c is α and $|b\times c|=\sqrt{15}$

$|b|\ |c|\sin\alpha=\sqrt{15}$

$\sin\alpha=\frac{\sqrt{15}}{4}$; $\therefore \cos\alpha=\frac{1}{4}$

$b-2c=\lambda a \Rightarrow |b-2c|^2=\lambda^2|a|^2$

$|b|^2+4|c|^2-4.b.c=\lambda^2|a|^2$

$16+4-4\{|b||c|\cos\alpha\}=\lambda^2$

$16+4-4\times 4\times 1\times\frac{1}{4}=\lambda^2 \Rightarrow \lambda^2=16 \Rightarrow \lambda=\pm 4$

Q.21 (3)

$$\Delta=\frac{1}{2}|a\times b|=\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & -1\\ 1 & 3 & 1\end{matrix}\right\|=\frac{1}{2}|(5\hat{i}-4\hat{j}+7\hat{k})|$$

$$\Delta=\frac{1}{2}\sqrt{25+16+49}=\frac{1}{2}\sqrt{90}=\frac{3}{2}\sqrt{10}.$$

Q.22 (3)

$a.(b\times c)=0$ or $(a\times b).c=0.$

Q.23 (1)

Let $a=3\hat{i}-2\hat{j}-\hat{k}$, $b=2\hat{i}+3\hat{j}-4\hat{k}$, $c=-\hat{i}+\hat{j}+2\hat{k}$

and $d=4\hat{i}+5\hat{j}+\lambda\hat{k}$.

Since the points are coplanar,

So, $[d\,b\,c]+[d\,c\,a]+[d\,a\,b]=[a\,b\,c]$

$$\Rightarrow\begin{vmatrix}4 & 5 & \lambda\\ 2 & 3 & -4\\ -1 & 1 & 2\end{vmatrix}+\begin{vmatrix}4 & 5 & \lambda\\ -1 & 1 & 2\\ 3 & -2 & -1\end{vmatrix}+\begin{vmatrix}4 & 5 & \lambda\\ 3 & -2 & -1\\ 2 & 3 & -4\end{vmatrix}$$

$$=\begin{vmatrix}3 & -2 & -1\\ 2 & 3 & -4\\ -1 & 1 & 2\end{vmatrix}$$

$$\Rightarrow 40+5\lambda+37-\lambda+94+13\lambda=25 \Rightarrow \lambda=\frac{-146}{17}.$$

Q.24 (4)

Volume of cube $=[a\,b\,c]$

$$=\begin{vmatrix}12 & 4 & 3\\ 8 & -12 & -9\\ 33 & -4 & -24\end{vmatrix}=12\begin{vmatrix}12 & 1 & 1\\ 8 & -3 & -3\\ 33 & -1 & -8\end{vmatrix}=3696.$$

Q.25 (2)

$i.(\hat{j}\times\hat{k})+\hat{j}.(\hat{k}\times\hat{i})+\hat{k}.(\hat{i}\times\hat{j})=\hat{i}.\hat{i}+\hat{j}.\hat{j}+\hat{k}.\hat{k}=3.$

Q.26 (2)

Vectors $i+3\hat{j}-2\hat{k};\ 2\hat{i}-\hat{j}+4\hat{k}$ and $3\hat{i}+2\hat{j}+x\hat{k}$. We know that as the vectors are coplanar, therefore

$$\begin{vmatrix}1 & 3 & -2\\ 2 & -1 & 4\\ 3 & 2 & x\end{vmatrix}=0$$

$\Rightarrow 1(-x-8)-3(2x-12)-2(4+3)=0$

$\Rightarrow -x-8-6x+36-14=0 \Rightarrow 7x=14 \Rightarrow x=2$

Q.27 (1)

$[a\times b\ \ b\times c\ \ c\times a]=(a\times b).[(b\times c)\times(c\times a)]$

$=(a\times b).([b\,c\,a]c-[b\,c\,c]a)=(a\times b).([b\,c\,a]c-0)$

$=[b\,c\,a][a\,b\,c]=[a\,b\,c][a\,b\,c]=4.4=16.$

Q.28 (3)

Let four points A,B,C,D represent the given points

So, $\overrightarrow{AB}=-\hat{i}-\hat{j}+4\hat{k},\ \overrightarrow{BC}=2\hat{i}+2\hat{j}-5\hat{k},$

$\overrightarrow{CD}=-2\hat{i}-(\lambda+4)\hat{j}+3\hat{k}$

From the condition, $[\overrightarrow{AB}\ \overrightarrow{BC}\ \overrightarrow{CD}]=0$

$$\Rightarrow \begin{vmatrix}-1 & -1 & 4\\ 2 & 2 & -5\\ -2 & -(\lambda+4) & 3\end{vmatrix}=0$$

$\Rightarrow$

$-1[2.3-5(\lambda+4)]+1[6-10]+4[-2(\lambda+4)+4]=0$

$\Rightarrow \lambda=-2$

Q.29 (1)

$b\times c$ is a vector perpendicular to $b,c.$ Therefore, $a\times(b\times c)$ is a vector again in plane of $b,c.$

Q.30 (3)

Let $a=x\hat{i}+y\hat{j}+z\hat{k}$

$\hat{i}\times(a\times\hat{i})+\hat{j}\times(a\times\hat{j})+\hat{k}\times(a\times\hat{k})$

$=(\hat{i}.\hat{i})a-\hat{i}(a.\hat{i})+(\hat{j}.\hat{j})a-\hat{j}(a.\hat{j})+(\hat{k}.\hat{k})a-\hat{k}(a.\hat{k})$

$=3a-a=2a.$

Q.31 (1)

$(a\times b)\times c=(a.c)b-(b.c)a$

$=(3+2+4)(2\hat{i}+\hat{j}-\hat{k})-(2-2-2)(3\hat{i}-\hat{j}+2\hat{k})$

$=18\hat{i}+9\hat{j}-9\hat{k}+6\hat{i}-2\hat{j}+4\hat{k}\ =24\hat{i}+7\hat{j}-5\hat{k}$

Q.32 (1)

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & -3\\ 2 & 1 & -1\end{vmatrix}=\hat{i}(2+3)-\hat{j}(-1+6)+\hat{k}(1+4)$$

$=5\hat{i}-5\hat{j}+5\hat{k}$

$$\text{Now } (\vec{A}\times\vec{B})\times\vec{C}=\begin{vmatrix}i & j & k\\ 5 & -5 & 5\\ 1 & 3 & -2\end{vmatrix}$$

$=\hat{i}(10-15)-\hat{j}(-10-5)+\hat{k}(15+5)$

$=-5\hat{i}+15\hat{j}+20\hat{k}=5(-\hat{i}+3\hat{j}+4\hat{k})$

Q.33 (4)

Required distance

$$=\left|\frac{d-a.n}{|\,n\,|}\right|=\left|\frac{5-(2\hat{i}-2\hat{j}+3\hat{k}).(\hat{i}+5\hat{j}+\hat{k})}{\sqrt{1+25+1}}\right|$$

$$=\left|\frac{5-(2-10+3)}{\sqrt{27}}\right|=\frac{10}{3\sqrt{3}}$$

Q.34 (1)

Let the equation of plane is

$a(x+1)+b(y+2)+c(z-0)=0$ …..(i)

As it passes through (2, 3, 5)

so, $3a+5b+5c=0$ …..(ii)

also, $2a+5b-c=0$ …..(iii)

$\therefore\ \dfrac{a}{-5-25}=\dfrac{b}{10+3}=\dfrac{c}{15-10}$

$\therefore\ \dfrac{a}{-30}=\dfrac{b}{13}=\dfrac{c}{5}$

Hence equation of plane is, $-30x+13y+5z=4$

or $r.(-30\hat{i}+13\hat{j}+5\hat{k})=4$

Q.35 (2)

The line of intersection of the planes $r.(\hat{i}-3\hat{j}+\hat{k})=1$ and $r.(2\hat{i}+5\hat{j}-3\hat{k})=2$ is perpendicular to each of the normal vectors $n_1=\hat{i}-3\hat{j}+\hat{k}$ and $n_2=2\hat{i}+5\hat{j}-3\hat{k}$

$\therefore$ It is parallel to the vector

$n_1\times n_2=(\hat{i}-3\hat{j}+\hat{k})\times(2\hat{i}+5\hat{j}-3\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 2 & 5 & -3 \end{vmatrix} = 4i + 5j + 11k$$

Q.36 (2)

Here d = 8 and $n = 2\hat{i} + \hat{j} + 2\hat{k})$

$$\therefore\ \hat{n} = \frac{n}{|n|} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{4+1+4}} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

Hence, the required equation of the plane is

$$r.\left(\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right) = 8 \text{ or } r.(2\hat{i} + \hat{j} + 2\hat{k}) = 24 .$$

Q.37 (3)

The given lines are $r = a_1 + \lambda b_1, r = a_2 + \mu b_2$

where $a_1 = 3\hat{i} - 2\hat{j} - 2\hat{k},\ b_1 = i$

$a_2 = \hat{i} - \hat{j} + 2\hat{k},\quad b_2 = \hat{j}$

$|b_1 \times b_2| = |\hat{i} \times \hat{j}| = |\hat{k}| = 1$

Now, $[(a_2 - a_1)\ b_1\ b_2] = (a_2 - a_1).(b_1 \times b_2)$

$= (-2\hat{i} + \hat{j} + 4\hat{k})(\hat{k}) = 4$

$\therefore$ Shortest distance

$$= \frac{[(a_2 - a_1)(b_1 - b_2)]}{|b_1 \times b_2|} = \frac{4}{1} = 4 .$$

Q.38 (2)

The equation of a plane parallel to the plane

$r.(4\hat{i} - 12\hat{j} - 3\hat{k}) - 7 = 0$ is $r.(4\hat{i} - 12\hat{j} - 3\hat{k}) + \lambda = 0$

This passes through $2\hat{i} - \hat{j} - 4\hat{k}$

Therefore, $(2\hat{i} - \hat{j} - 4\hat{k}).(4\hat{i} - 12\hat{j} - 3\hat{k}) + \lambda = 0$

$\Rightarrow 8 + 12 + 12 + \lambda = 0 \Rightarrow \lambda = -32$

So, the required plane is $r.(4\hat{i} - 12\hat{j} - 3\hat{k}) - 32 = 0$

Q.39 (1)

The vector equation of a plane through the line of intersection of the planes $r.(\hat{i} + 3\hat{j} - \hat{k}) = 0$ and $r.(\hat{j} + 2\hat{k}) = 0$ can be written as

$(r.(\hat{i} + 3\hat{j} - \hat{k})) + \lambda(r.(\hat{j} + 2\hat{k})) = 0$ .....(i)

This passes through $2\hat{i} + \hat{j} - \hat{k}$

$\therefore\ (2\hat{i} + \hat{j} - \hat{k}).(\hat{i} + 3\hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j} - \hat{k}).(\hat{j} + 2\hat{k}) = 0$

or $(2 + 3 + 1) + \lambda(0 + 1 - 2) = 0 \Rightarrow \lambda = 6$

Put the value of λ in (i) we get

$r.(\hat{i} + 9\hat{j} + 11\hat{k}) = 0$ which is the required plane.

Q.40 (2)

The equation of a line passing through the points $A(\hat{i} - \hat{j} + 2\hat{k})$ and $B(3\hat{i} + \hat{j} + \hat{k})$ is

$$r = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + \hat{j} + \hat{k})$$

The position vector of any point P which is a variable point on the line, is

$(\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + \hat{j} + \hat{k})$

$\overrightarrow{AP} = \lambda(3\hat{i} + \hat{j} + \hat{k}) \Rightarrow |\overrightarrow{AP}| = \lambda\sqrt{11}$

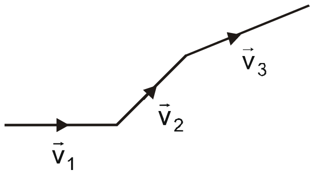
Now, if $\lambda\ \sqrt{11} = 3\sqrt{11}$ i.e., $\lambda = 3$ then the position vector of the point P is $10\hat{i} + 2\hat{j} + 5\hat{k}$.

If $\lambda\sqrt{11} = -3\sqrt{11},$ i.e., $\lambda = -3$ then the position vector of the point P is $-8\hat{i} - 4\hat{j} - \hat{k}$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

Clearly triangle is not possible as $v_1 + v_2 + v_3 \neq 0$



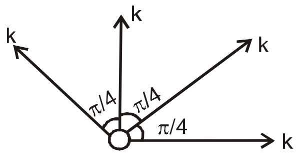
Since $\vec{v}_3 = \vec{v}_1 + \vec{v}_2$

Hence $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are coplaner

Q.2 (3)

These forces can be written in terms of vector as

$k\hat{i},\ \frac{k}{\sqrt{2}}\hat{i} + \frac{k}{\sqrt{2}}\hat{j},\ k\hat{j}$ and $-\frac{k}{\sqrt{2}}\hat{i} + \frac{k}{\sqrt{2}}\hat{j}$



Resultant = $k\hat{i} + (k + \sqrt{2}k)\hat{j}$

magnitude = $\sqrt{k^2 + (k + \sqrt{2}k)^2} = k\sqrt{4 + 2\sqrt{2}}$

Q.3 (2)

Before rotation $\vec{a} = 2p\hat{i} + \hat{j}$

after rotation $\vec{a} = (p+1)\hat{i}' + \hat{j}'$

Since length of vector remains unaltered

$\sqrt{4p^2 + 1} = \sqrt{(p+1)^2 + 1}$

$\Rightarrow \quad 4p^2 = (p + 1)^2 \quad \Rightarrow \; p + 1 = \; \pm \; 2p$

$\Rightarrow \quad p = 1 \;\text{ or }\; -\dfrac{1}{3}$

Q.4 (3)

$\vec{a} = (2\sqrt{2}, -1, 4) \;\; |\vec{b}| = 10$

$\vec{b} = \lambda\vec{a}$

$|\vec{b}|^2 = \lambda^2\,|\vec{a}|^2$

$100 = \lambda^2\,(8 + 1 + 16)$

$\lambda^2 = 4 \;\Rightarrow \lambda = \pm 2$

$2\vec{a} \pm \vec{b} = 0$

Q.5 (4)

$\overline{AB} = -6\,\hat{i} - 10\,\hat{j} + 3\hat{k}$

$\overline{AD} = -2\,\hat{i} - 5\,\hat{j} - 2\hat{k}$

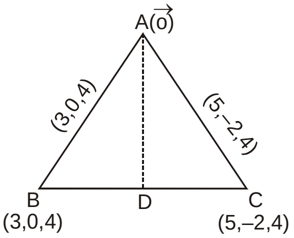
$\overline{AB} \cdot \overline{AD} \neq 0$

so not a square or rectangle $\left|\overline{AB}\right| \neq \left|\overline{AD}\right|$ so not a rhombus.

Q.6 (3)

$\overrightarrow{AB} = (3, 0, 4)$

$\overrightarrow{AC} = (5, -2, 4)$

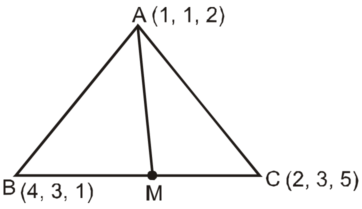


Let $\vec{A}$ be origin.
D is the mid
point of BC
D(4, –1, 4)

$\overrightarrow{AD} = (4, -1, 4)$

$|\overrightarrow{AD}| = \sqrt{16 + 1 + 16} = \sqrt{33}$

Q.7 (4)



$AB = \sqrt{9 + 4 + 1} = \sqrt{14}$

$AC = \sqrt{1 + 4 + 9} = \sqrt{14}$

$M \equiv (3, 3, 3)$

$\overrightarrow{AM} = 2\hat{i} + 2\hat{j} + \hat{k}$

Q.8 (4)

$\vec{a}.(\vec{b} + \vec{c}) = 0$ ...(i)

$\vec{b}.(\vec{c} + \vec{a}) = 0$ ...(ii)

$\vec{c}.(\vec{a} + \vec{b}) = 0$ ....(iii)

Add all equation $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$

$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 0}$

$= \sqrt{9 + 16 + 25} = 5\sqrt{2}$

Q.9 (4)

$\vec{A}$ (2, –1, –1), $\vec{B}$ = (1, –3, –5),

$\vec{C}$ = (a, –3, –1)

$\overrightarrow{AC} \,.\, \overrightarrow{CB} = 0 \Rightarrow (a - 2, -2, 0) \,.\, (a - 1, 0, 4) = 0$

$(a - 1)\,(a = 2) = 0 \Rightarrow a = 1$ and 2

Q.10 (1)

$\vec{F}_1 = (4, 1, -3) \;\; \vec{F}_2 = (3, 1, -1)$

$d\vec{s} = (5, 4, 1) - (1, 2, 3) = (4, 2, -2)$

work done = $\vec{F}_1.d\vec{s} + \vec{F}_2.d\vec{s} = 24 + 16 = 40$

Q.11 (3)

$\vec{a} = \hat{i} - \hat{j},\; \vec{b} = \hat{i} + \hat{j},\; \vec{c} = \hat{i} + 3\hat{j} + 5\hat{k} \;\Rightarrow\; \vec{n} = \hat{k} \;\Rightarrow$

$|\vec{c}.\vec{n}| = 5$

Q.12 (2)

$|\vec{a} - \vec{b}| = 8 \Rightarrow |a|^2 + |b|^2 - 2\vec{a}.\vec{b} = 64$ ..(i)

$|\vec{a} + \vec{b}| = 10 \Rightarrow |a|^2 + |b|^2 + 2\vec{a}.\vec{b} = 100$ ..(ii)

Add (i) and (ii) equation

$2|a|^2 + 2|b|^2 = 164$

$|b|^2 = 82 - 25$

$|b| = \sqrt{57}$

Q.13 (1)

Diagonals are $\vec{a} + \vec{b} = (3, 0, 0)$

and $\vec{a} - \vec{b} = (1, 2, 2)$

$\cos\theta = \dfrac{(\vec{a}+\vec{b}).(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}|\ \ |\vec{a}-\vec{b}|} = \dfrac{3}{3.3} = \dfrac{1}{3}$

$\theta = \cos^{-1}\left(\dfrac{1}{3}\right)$

Q.14 (4)

$|\vec{u}| = 1;\ \ |\vec{v}| = 1$

$|\ 2\vec{u} \times 3\vec{v}\ | = 1$

$|\ \vec{u} \times \vec{v}\ | = \dfrac{1}{6}$

$|\ u\ |\ |\ v\ | \sin\theta = \dfrac{1}{6} = \boxed{\sin\theta = \dfrac{1}{6}}$

As θ is acute angle than only one value possible

Q.15 (2)

$\vec{u} = \vec{a}-\vec{b}$, $\vec{v} = \vec{a}+\vec{b}$, $|\vec{a}| = |\vec{b}| = 2$

$|\vec{u}\times\vec{v}| = |(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})|$

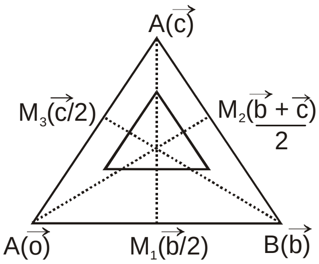
$= |(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})|$

$= 2\ |\ \vec{a}\times\vec{b}\ | = 2\ |\ a\ |\ |\ b\ | \sin\theta$

$= 2\ |\ a\ |\ |\ b\ |\ \sqrt{\dfrac{|a|^2|b|^2-(\vec{a}.\vec{b})^2}{|a||b|}} = 2\sqrt{16-(\vec{a}.\vec{b})^2}$

Q.16 (2)

$C_1\left(\dfrac{\vec{b}/2 + 3\vec{c}}{4}\right)$



$A_1\left(\dfrac{\vec{b}+\vec{c}/2 + 0}{4}\right)$

$B_1\left(\dfrac{\vec{c}/2 + 3\vec{b}}{4}\right)$

Area of $\Delta A_1B_1C_1 = \dfrac{1}{2}|\ \overrightarrow{A_1B_1}\times\overrightarrow{A_1C_1}\ |$

Area of $\Delta ABC = \dfrac{1}{2}|\vec{b}\times\vec{c}\ |$

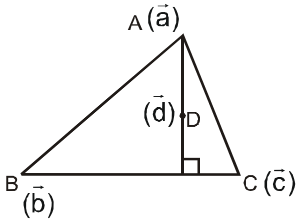
Ratio $= \dfrac{\text{Area of } \Delta A_1B_1C_1}{\text{Area of } \Delta ABC} = \dfrac{25}{64}$.

Q.17 (3)

Given

$(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c}) = (\vec{b}-\vec{d}).(\vec{c}-\vec{a}) = 0$

$\Rightarrow \overrightarrow{DA}.\overrightarrow{CB} = 0\ \ \&\ \ \overrightarrow{DB}.\overrightarrow{AC} = 0$



AD ⊥ BC & BD ⊥ AC

Hence D is orthocentre.

Q.18 (2)

A vector normal to plane is $(\vec{a}-\vec{b})\times(\vec{c}-\vec{b})$

$= \vec{a}\times\vec{c} - \vec{a}\times\vec{b} - \vec{b}\times\vec{c} = -(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})$

unit vector $= \pm\ \dfrac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$

Q.19 (1)

$|\ \vec{e}_1-\vec{e}_2\ |^2 < 1 \Rightarrow\ \ \vec{e}_1^{\ 2}+\vec{e}_2^{\ 2}-2\vec{e}_1.\vec{e}_2 < 1$

$\Rightarrow\ \ 1\ +1 - 2\cos(2\theta) < 1$

$\Rightarrow\ \ 2\cos2\theta > 1\ \ \Rightarrow\ \ \cos2\theta > \dfrac{1}{2}$

$2\theta \in \left[0, \dfrac{\pi}{3}\right)\ \ \Rightarrow\ \ \theta \in \left[0, \dfrac{\pi}{6}\right)$

Q.20 (4)

$\vec{a} = (1, x, 3)$ $\qquad\qquad \cos\theta = \dfrac{11}{14}$

$\vec{b} = (4, 4x-2, 2)$

$\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$ $\qquad\qquad |\vec{b}| = 2\ |\vec{a}\ |$

$\dfrac{11}{14} = \dfrac{4+x(4x-2)+6}{2|a|^2} \Rightarrow x = 2 \text{ and } x = -\dfrac{20}{17}$.

Q.21 (2)

$\vec{a} = (-2, 1, 1)$, $\vec{b} = (1, 5, 0)$, $\vec{c} = (4, 4, -2)$

$\vec{d} = 3\vec{a} - 2\vec{b}$

$= 3(-2, 1, 1) - 2(1, 5, 0)$

$= (-6, 3, 3) - 2(2, 10, 0) = (-8, -7, 3)$

Projection $= |\vec{d}|\cos\theta$

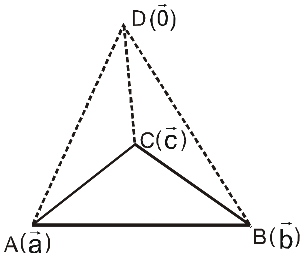
$$= |\vec{d}|\frac{\vec{d}.\vec{c}}{|\vec{d}||\vec{c}|} = \frac{\vec{d}.\vec{c}}{|\vec{c}|} = \frac{-31-28-6}{\sqrt{16+16+4}} = \frac{-66}{6} = -$$

11.

Q.22 (1)

$\vec{a}_1 = \overrightarrow{AC}\times\overrightarrow{AB} = (\vec{c}-\vec{a}) \times (\vec{b}-\vec{a})$

$\vec{a}_2 = \overrightarrow{DB}\times\overrightarrow{DC} = \vec{b}\times\vec{c}$



$\vec{a}_3 = \overrightarrow{DC}\times\overrightarrow{DA} = \vec{c}\times\vec{a}$

$\vec{a}_4 = \overrightarrow{DA}\times\overrightarrow{DB} = \vec{a}\times\vec{b}$

Q.23 (4)

$\vec{a}.\vec{b} = 0 \Rightarrow x - y + 2 = 0$ ... (1)

$\vec{a}.\vec{c} = 4 \Rightarrow x + 2y = 4$ ... (2)

$\Rightarrow x = 0, \quad y = 2$

Hence $\vec{a} = 2\hat{j} + 2\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 0 & 2 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 8 = |\vec{a}|^2$$

Q.24 (4)

$\vec{a} = (1, 1, 1)$ $\vec{b} = (1, 1, 1)$

$\vec{c} = (2, -3, 0)$

$\vec{v} = \vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})c = (-7, 8, -1)$

$$\vec{v} = \frac{(-7, 8, -1)}{\sqrt{114}}$$

Reqd. Vector $= \dfrac{3}{\sqrt{114}}(-7\hat{i} + 8\hat{j} - \hat{k})$

Q.25 (D)

$(\vec{a}\times\vec{b})\times\vec{c} = \dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$(\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\,\vec{a} = \dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\cos\theta = -\dfrac{1}{3} \Rightarrow \sin\theta = \dfrac{2\sqrt{2}}{3}$

Q.26 (4)

$\vec{a}\times(2\vec{b}\times\vec{c}) = \vec{b} \Rightarrow 2\left[(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}\right] = \vec{b}$

$\Rightarrow \vec{a}.\vec{c} = \dfrac{1}{2}$ & $\vec{a}.\vec{b} = 0$

Q.27 (1)

$\vec{a} \parallel (\vec{b}\times\vec{c}) \Rightarrow$ $\vec{a} = \lambda(\vec{b}\times\vec{c})$

also $(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c}) = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{c} \end{vmatrix} = \begin{vmatrix} \vec{a}.\vec{a} & 0 \\ 0 & \vec{b}.\vec{c} \end{vmatrix}$

$= |\vec{a}|^2\,(\vec{b}.\vec{c})$

Q.28 (3)

$[(\vec{a}+2\vec{b}-\vec{c}),\ (\vec{a}-\vec{b}),\ (\vec{a}-\vec{b}-\vec{c})]$

$(\vec{a}+2\vec{b}-\vec{c})\ .\ [(\vec{a}-\vec{b})\times(\vec{a}-\vec{b}-\vec{c})]$

$(\vec{a}+2\vec{b}-\vec{c})\ .\ [(\vec{b}-\vec{a})\times\vec{c}]$

$(\vec{a}+2\vec{b}-\vec{c})\ .\ [\vec{b}\times\vec{a}-\vec{a}\times\vec{c}]$

$= [\vec{a}\ \ \vec{b}\ \ \vec{c}] - 0 + 0 + 2[\vec{a}\ \ \vec{b}\ \ \vec{c}] - 0 + 0$

$3[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

Q.29 (3)

$\vec{r}.(\vec{a}+\vec{b}+\vec{c}) = 0$

$\Rightarrow \ell[\vec{a}\ \vec{b}\ \vec{c}] + m[\vec{a}\ \vec{b}\ \vec{c}] + n[\vec{a}\ \vec{b}\ \vec{c}] = 0$

or $(\ell + m + n)\ [\vec{a}\ \vec{b}\ \vec{c}] = 0$

or $\ell + m + n = 0$

Q.30 (3)

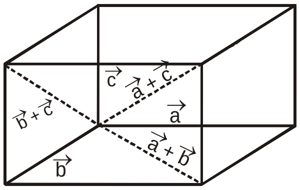
Altitude from D $= \dfrac{\text{Volume of Tetrahedron}}{\text{Area of Face ABC}}$

$$= \frac{[\overline{AD}\ \ \overline{AC}\ \ \overline{AB}]}{\frac{1}{2}|\overline{AB}\times\overline{AC}|} = 11$$

Q.31 (1)

$V_{Old} = [\vec{a}\ \ \vec{b}\ \ \vec{c}]$

$V_{New} = [\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] \quad = 2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$



so $m = 2$

Q.32 (3)

$$\begin{vmatrix} \vec{a}\,.\,\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = [\vec{a}\ \vec{b}\ \vec{c}]^2 = 4^2 = 16$$

Q.33 (3)

$\left[\vec{b}\ \vec{c}\ \vec{a}\right] < 0$

$$\begin{vmatrix} 2 & 2x & 1 \\ 1 & 0 & 1 \\ x & 12 & -1 \end{vmatrix} < 0$$

$2(0 - 12) - 2x\,(-1 - x) + 1\,(12) < 0$

or $-24 + 2x + 2x^2 + 12 < 0$

$\Rightarrow \quad x^2 + x - 6 < 0$

$\Rightarrow x \in (-3, 2)$

Q.34 (3)

$\vec{a}, \vec{b}, \vec{c}$ are non-coplaner $\Rightarrow \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] \neq 0$

$\left[\vec{a}+2\vec{b}+3\vec{c}\quad \lambda\vec{b}+4\vec{c}\quad (2\lambda-1)\vec{c}\right]$

$= (\vec{a}+2\vec{b}+3\vec{c})\ .\ [(\lambda\vec{b}+4\vec{c})\times(2\lambda-1)\vec{c}]$

$= \lambda\,(2\lambda - 1)\ (\vec{a}+2\vec{b}+3\vec{c})\ .\ (\vec{b}\times\vec{c})$

$= \lambda\,(2\lambda - 1)\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]$

Q.35 (2)

$[\vec{u}\quad \vec{v}\quad \vec{w}] \neq 0$

$(\vec{u}+\vec{v}-\vec{w})\ .\,[(\vec{u}-\vec{v}) \times (\vec{v}-\vec{w})]$

$(\vec{u}+\vec{p})\ .\ [(\vec{u} - \vec{v}) \times \vec{p}]$

$(\vec{u}+\vec{p})\ .\ [\vec{u} \times \vec{p} - \vec{v} \times \vec{p}]$

$= -\,\vec{u}\ .\ (\vec{v}\times\vec{p}) \ = -\,\vec{u}\ .\ (\vec{v}\times(\vec{v}-\vec{w}))$

$= \vec{u}\ .\ (\vec{v}\times\vec{w})$

Q.36 (3)

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Given $\vec{c}.\vec{a} = 0$ & $\vec{c}.\vec{b} = 0$ $= \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

$$= \begin{vmatrix} |a|^2 & |a||b|\cos\frac{\pi}{6} & 0 \\ |a||b|\cos\frac{\pi}{6} & |b|^2 & 0 \\ 0 & 0 & |c|^2 \end{vmatrix}$$

$= |c|^2\ [\ |a|^2 + |b|^2 - |a|^2\,|b|^2 \cos^2 \frac{\pi}{6}\ ]$

$= |c|^2\,|a|^2\,|b|^2 \left[1 - \frac{3}{4}\right] = \frac{1}{4}\ |c|^2\,|a|^2\,|b|^2$

$= \frac{1}{4}\ (a_1^2 + a_2^2 + a_3^2)\ (b_1^2 + b_2^2 + b_3^2)$

Q.37 (3)

$$[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \cos\theta & \cos\theta \\ \cos\theta & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix}$$

Q.38 (1)

Assume $\vec{b} = \hat{i},\ \vec{c} = \hat{j}$ and $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{a}.\vec{b} = 1,\ \vec{a}.\vec{c} = 1$

$\vec{b}+\vec{c}+\vec{k} = \hat{i}+\hat{j}+\hat{k} = \vec{a}.$

Q.39 (1)

$$\begin{vmatrix} m & m+1 & m+8 \\ m+3 & m+4 & m+5 \\ m+6 & m+7 & m+8 \end{vmatrix}$$

$R_1 \to R_2 - R_1$

$R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 3 & 3 & -3 \\ -3 & -3 & -3 \\ m+6 & m+7 & m+8 \end{vmatrix}$$

$$= -9 \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ m+6 & m+7 & m+8 \end{vmatrix}$$

$= -9[m + 8 - m - 7] - 1[m+8-m-6] - 1[m+7-m-6]$

$= -9 - 2 - 1 = -12.$

Q.40 (3)

$\vec{a} = \hat{i} + \hat{j}$ $\qquad$ $\vec{b} = 2\hat{i} - \hat{k}$

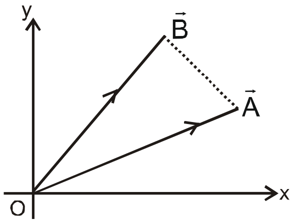
$\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ .....(i)

$\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ .....(ii)

Add (i) & (ii)

$\vec{r} \times (\vec{a} \times \vec{b}) = 0 \Rightarrow \vec{r} = (\vec{a} + \vec{b}) = (3, 1, -1)$

Q.41 (3)



$\overrightarrow{OA} \times \overrightarrow{OB}$ = a fixed vector

$\Rightarrow |\overrightarrow{OA} \times \overrightarrow{OB}|$ = const. number

$\Rightarrow \Delta OAB$ = const.

$\Rightarrow$ B is on the line || to base OA

Q.42 (3)

Let D is of c on line

$AC = \sqrt{(1)^2 + (-2)^2 + (1)^2}$

AD = proj. of AC on AD

$= \frac{1(6) + (-2)(-3) + 1(2)}{7}$

AD = 2

So shortest distance $(CD)^2 = (AC)^2 - (AD)^2$

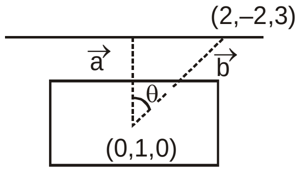
$= 6 - 4 = 2$

$CD = \sqrt{2}$.

Q.43 (3)

$\vec{r} = (2, -2, 3) + \lambda(1, -1, 4)$

$\vec{r} \cdot (1, 5, 1) = 5$



$\vec{L} \cdot \vec{n} = 1 - 5 + 4 = 0$

So line and plane

are parallel.

Let a point on the plane (0, 1, 0)

distance = | b | cos θ

$= |b| \frac{\vec{a}.\vec{b}}{|a||\vec{b}|} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = \left|\frac{2 - 15 + 3}{\sqrt{27}}\right| = \frac{10}{3\sqrt{3}}$

Q.44 (2)

Equation of Altitude or plane is

$\vec{r} = i - 2\hat{i} + 2\hat{k} + \lambda(2\hat{i} + 3\hat{j})$

Let a point of line

$x = 1 + 2\lambda$

$y = -2 + 3\lambda$

$z = 2 - 2\lambda$

Put there point in the of plane

$((1 + 2\lambda)\hat{i} + (-2 + 3\lambda)\hat{j} + (2 - 2\lambda)\hat{k})$

$\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right) + 312 = 0$

and find l

Put the value of l and get two point

Be cause Intersection point is nidpoint of Requred point and given point

Q.45 (1)

$(2\hat{i} + \hat{j} + 2\hat{k}).(3\hat{i} - 2\hat{j} - m\hat{k}) = 0$

$= 6 - 2 - 2m = 0$ or $m = 2$

Q.46 (1)

Normal Vector $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 5(\hat{i} - \hat{j} - \hat{k})$

Let $\vec{A} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$. If θ is the angle between vector $\vec{A}$ and plane then 90 – θ will be the angle between normal and plane

$\cos(90 - \theta) = \frac{5\alpha - 5\beta - 5\gamma}{5\sqrt{3}\sqrt{\alpha^2 + \beta^2 + \gamma^2}}$

$$\sin^2\theta = \frac{(\alpha-\beta-\gamma)^2}{3(\alpha^2+\beta^2+\gamma^2)} \Rightarrow \boxed{\beta\gamma = \alpha(\beta+\gamma)}\ .$$

Q.47 (2)

$$[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2 \neq 0$$

$$[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}] = 2(\vec{a}\ \vec{b}\ \vec{c}] \neq 0$$

Q.48 (1)

We have $\vec{a}\perp\vec{b} \Rightarrow \vec{a},\vec{b},\vec{a}\times\vec{b}$ are linearly independent.

$\vec{v}$ can be expressed uniquely in terms of $\vec{a},\vec{b}$ and $\vec{a}\times\vec{b}$.

$$\vec{v} = x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b})$$

Given $\vec{a}\cdot\vec{b}=0$, $\vec{v}\cdot\vec{a} = 0$, $\vec{v}\cdot\vec{b} = 1$, $[\vec{v}\ \vec{a}\ \ \vec{b}] = 1$

$$\vec{v}\cdot\vec{a} = x\vec{a}^2 = 0 \Rightarrow \quad x = 0$$

$$\vec{v}\cdot\vec{b} = x\vec{a}\cdot\vec{b}+y\vec{b}^2+z(b)\cdot(\vec{a}\times\vec{b}) = 1$$

$$yb^2 = 1 \qquad \Rightarrow \qquad y = \frac{1}{b^2}$$

$$\vec{v}\cdot(\vec{a}\times\vec{b}) = x.0+y.0+z|\vec{a}\times\vec{b}|^2 = 1$$

$$z = \frac{1}{|\vec{a}\times\vec{b}|^2}$$

$$\vec{v} = \frac{1}{|\vec{b}|^2}\vec{b}+\frac{1}{|\vec{a}\times\vec{b}|^2}(\vec{a}\times\vec{b})$$

Q.49 (2)

$$\vec{A}\ .\ \vec{X} = C$$

$$\vec{A}\times\vec{X} = \vec{B}$$

take cross with $\vec{A}$

$$\vec{A}\times(\vec{A}\times\vec{X}) = \vec{A}\times\vec{B}$$

$$\vec{X} = \frac{C\vec{A}-\vec{A}\times\vec{B}}{|\vec{A}|^2}$$

Q.50 (3)

$\vec{A}.\vec{B} = \vec{A}.\vec{C}$ and $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$

$$\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$$

$$\vec{A}\times(\vec{A}\times\vec{B}) = \vec{A}\times(\vec{A}\times\vec{C})$$

$$(\vec{A}.\vec{B})\vec{A}-|A|^2\ \vec{B} = (\vec{A}\times\vec{C})\vec{A}-|A|^2\ \vec{C}$$

$$\vec{B} = \vec{C}$$

JEE-ADVANCED OBJECTIVE QUESTIONS

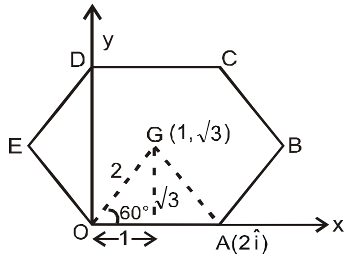
Q.1 (C)

$$\vec{b} = \lambda(2\sqrt{2}\,\hat{i}-\hat{j}+4\hat{k});\ |\vec{b}| = 10$$

$$\Rightarrow |\lambda|\sqrt{8+1+16} = 10 \qquad \Rightarrow \lambda = \pm 2$$

$$\Rightarrow \vec{b} = \pm 2\vec{a}$$

Q.2 (C)



$$G \equiv (\hat{i}+\sqrt{3}\,\hat{j})$$

Let Position vector of P is $\vec{p}$

$$\because\ \overrightarrow{GP}\,||\,\hat{k}$$

then $\vec{p} - (\hat{i}+\sqrt{3}\,\hat{j}) = \lambda\hat{k}$

$$\Rightarrow \vec{p} = \hat{i}+\sqrt{3}\ \hat{j} + \lambda\hat{k}$$

also $\left|\overrightarrow{OP}\right| = 3$

$$\Rightarrow \sqrt{1+3+\lambda^2} = 3$$

$$\Rightarrow \lambda^2 = 5$$

$$\Rightarrow \lambda = \pm\sqrt{5}$$

$$\Rightarrow \vec{p} = \hat{i}+\sqrt{3}\ \hat{j}\pm\sqrt{5}\,\hat{k}$$

For positive Z-axis $\vec{p} = \hat{i}+\sqrt{3}\,\hat{j}+\sqrt{5}\ \hat{k}$

So $\overrightarrow{AP} = \vec{p}-2\hat{i} = -\ \hat{i}+\sqrt{3}\,\hat{j}+\sqrt{5}\ \hat{k}$

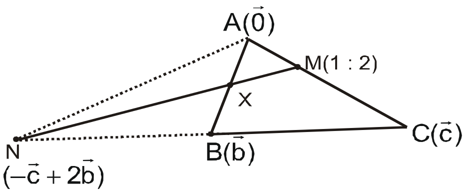
Q.3 (C)

Position vector of M $\equiv \dfrac{\vec{c}}{3}$

Position vector of N $\equiv (-\vec{c}+2\vec{b})$

$\therefore$ equation of line BC is $\vec{r} = \vec{b}+\lambda(\vec{b}-\vec{c})$

$\therefore$ equation of line AB is is $\vec{r} = \vec{0}+\mu\vec{b}$



$\therefore$ equation of line MN is $\vec{r} = \frac{\vec{c}}{3} + t\left(\frac{4\vec{c}}{3} - 2\right)$

$\Rightarrow \mu = -2t, \quad 0 = \frac{1}{3} + \frac{4}{3}t$

which gives $\mu = \frac{1}{2} \Rightarrow$ Position vector of X is $\frac{\vec{b}}{2}$.

Q.4 (D)

$\vec{a}.(\vec{b}+\vec{c}) = 0$, $\quad \vec{b}.(\vec{c}+\vec{a}) = 0$,

$\vec{c}.(\vec{a}+\vec{b}) = 0$

$\Rightarrow \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0, \quad \vec{b}.\vec{c} + \vec{b}.\vec{a} = 0, \quad \vec{c}.\vec{a} + \vec{c}.\vec{b} = 0$

$\Rightarrow \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$|\vec{a}+\vec{b}+\vec{c}|$

$= \sqrt{\vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})}$

$= \sqrt{9+16+25} = \sqrt{50}$

Q.5 (B)

Let $\overrightarrow{AB} = \sqrt{3}\,(\hat{a} \times \vec{b})$

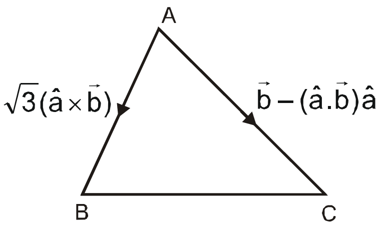
$\overrightarrow{AC} = \vec{b} - (\hat{a}\,.\,\vec{b})\hat{a}$

$\overrightarrow{AB}\,.\,\overrightarrow{AC} = 0$

$|\overrightarrow{AB}| = \sqrt{3}\,|\hat{a}||\vec{b}|\sin\theta = \sqrt{3}\,|\vec{b}|\sin\theta$

$|\overrightarrow{AC}|^2 = \vec{b}^2 + (\hat{a}.\,\vec{b})^2\hat{a}^2 - 2(\hat{a}.\vec{b})(\hat{a}.\,\vec{b})$

$= \vec{b}^2 + |\vec{b}|^2\cos^2\theta - 2|\vec{b}|^2\cos^2\theta$



$= |\vec{b}|^2\sin^2\theta$

$\Rightarrow |\overrightarrow{AC}| = |\vec{b}|\sin\theta$

$\therefore$ Δ ABC is right angled and ratio $\frac{|\overrightarrow{AB}|}{|\overrightarrow{AC}|} = \sqrt{3}$

angles are 90º, 60º, 30º

Q.6 (C)

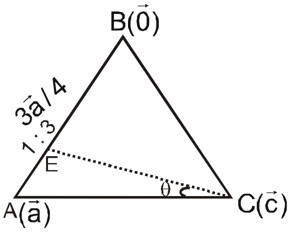
$|\overrightarrow{AB}| = |\overrightarrow{BC}| = 8$

or $|\vec{a}| = |\vec{c}| = 8$

CA = 12 $\Rightarrow |\vec{c} - \vec{a}| = 12$

$\Rightarrow \vec{c}^2 + \vec{a}^2 - 2\vec{c}.\vec{a} = 144$

$\Rightarrow$ 64 + 64 – 144 = $2\vec{c}.\vec{a}$ or $\vec{c}.\vec{a} = -8$



Consider $\overrightarrow{CE}.\overrightarrow{CA} = |\overrightarrow{CE}||\overrightarrow{CA}|\cos\theta$

or $\left(\frac{3\vec{a}}{4} - \vec{c}\right)\cdot(\vec{a} - \vec{c}) = \left|\frac{3}{4}\vec{a} - \vec{c}\right|$. 12 cosθ

$\Rightarrow \frac{3}{4}\vec{a}.\vec{a} - \frac{3}{4}\vec{a}.\vec{c} - \vec{c}.\vec{a} + \vec{c}.\vec{c} = \sqrt{112}$ 12 cosθ

or $\frac{3}{4}.64 - \frac{3}{4}(-8) - (-8) + 64 = \sqrt{112}$ 12 cosθ

$\Rightarrow$ 48 + 6 + 8 + 64 = $\sqrt{112}$ 12cosθ

$\Rightarrow \cos\theta = \frac{3\sqrt{7}}{8}$

Q.7 (B)

$\frac{1}{a} = A + (p-1)\,D$, $\frac{1}{b} = A + (q-1)\,D$,

$\frac{1}{c} = A + (r-1)\,D$

$\Rightarrow \frac{1}{a} - \frac{1}{b} = (p-q)\,D$ or $p - q = \frac{b-a}{abD}$ and so on

$\Rightarrow \quad \vec{u} = \frac{c-b}{bcD}\hat{i} + \frac{a-c}{caD}\hat{j} + \frac{b-a}{abD}\hat{k}$

Consider $\vec{u}.\vec{v} = \frac{c-b}{abcD} + \frac{a-c}{abcD} + \frac{b-a}{abcD} = 0$

Q.8 (C)

$|\vec{u} - \vec{v} + \vec{\omega}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + |\vec{\omega}|^2 - 2(\vec{u}.\vec{v} + \vec{v}.\vec{\omega} - \vec{\omega}.\vec{u})$

$= 14 - 2\ (\vec{u}.\vec{v} + \vec{v}.\vec{\omega} - \vec{\omega}.\vec{u})$

Given $\frac{\vec{u}.\vec{v}}{|\vec{u}|} = \frac{\vec{\omega}.\vec{u}}{|\vec{u}|} = \frac{\vec{\omega}.\vec{v}}{|\vec{u}|}$ and $\vec{\omega}.\vec{v} = 0$ & $\vec{\omega}.\vec{i} = 0$

$\vec{u}.\vec{v} = \vec{\omega}.\vec{u} = \frac{\vec{\omega}.\vec{v}}{|\vec{v}|}$

$|\vec{u} - \vec{v} + \vec{\omega}|^2 = 14 - 2\ (2\,\vec{\omega}.\vec{u})$

$|\vec{u} - \vec{v} + \vec{\omega}|^2 = 14$

$|\vec{u} - \vec{v} + \vec{\omega}| = \sqrt{14}$

Q.9 (D)

$\vec{u} = (1, 1, 0)$, $\vec{v} = (1, -1, 0)$, $\vec{w} = (1, 2, 3)$

$\vec{u}.\hat{n} = 0$, $\vec{v}.\hat{n} = 0$ then $\Rightarrow |\vec{w}.\hat{n}| = |\pm 3| = 3$

where $\hat{n} = \lambda(\vec{u} \times \vec{v}) \Rightarrow \hat{n} = -2\lambda\hat{k}$

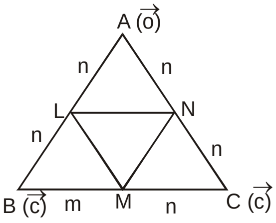
$|\hat{n}| = 1 \Rightarrow \lambda = \pm\frac{1}{2} \Rightarrow 2\lambda = \pm 1$

Q.10 (B)

Assume A $(\vec{0})$, B $(\vec{b})$ C $(\vec{c})$

Position vector of L, M, N

$L\left(\frac{m\vec{b}}{m+n}\right)\ N\left(\frac{m\vec{c}}{m+n}\right)\ M\left(\frac{m\vec{c}+n\vec{b}}{m+n}\right)$



Area of $\Delta ABC = \frac{1}{2}|\vec{c} \times \vec{b}|$

Area of $\Delta LMN = \frac{1}{2}|\overrightarrow{LN} \times \overrightarrow{LM}|$

$= \frac{1}{2}\left|\left(\frac{n\vec{c} - m\vec{b}}{m+n}\right) \times \left(\frac{m\vec{c} + n\vec{b} - m\vec{b}}{m+n}\right)\right|$

$= \frac{1}{2}\frac{1}{(m+n)^2}\left|(n(n-m) + m^2)\ (\vec{c} \times \vec{b})\right|$

$\frac{A(\Delta LMN)}{A(\Delta ABC)} = \frac{n^2 - mn + m^2}{(m+n)^2}$

Q.11 (A)

If the radius of circum centre = r

$|\overrightarrow{OA_i}| = r$ where i = 1, 2, 3, ... n

$\therefore \sum \overrightarrow{OA_i} \times |\overrightarrow{OA_i}| = \sum |\overrightarrow{OA_i}|\ |\overrightarrow{OA_{i+1}}| \sin \frac{2\pi}{n}\hat{n}$

$= \sum r^2 \sin\frac{2\pi}{n}\hat{n} = (n-1)r^2 \sin\frac{2\pi}{n}\hat{n}$

$= (n-1)\ (\overrightarrow{OA_1} \times \overrightarrow{OA_2}) = (1-n)\ (\overrightarrow{OA_2} \times \overrightarrow{OA_1})$

Q.12 (B)

Let $S(\vec{o})$, $P(\vec{a})$, $Q(\vec{b})$, $R(\vec{c})$

$= |\overrightarrow{PQ} \times \overrightarrow{RS} - \overrightarrow{QR} \times \overrightarrow{PS} + \overrightarrow{RP} \times \overrightarrow{QS}| = |(\vec{b} - \vec{a}) \times (-\vec{c}) - (\vec{c} - \vec{b}) \times (-\vec{a}) + (\vec{a} - \vec{c}) \times (-\vec{b})|$

$= 2\,|(\vec{c} \times \vec{b})| = 2\ (\vec{b} \times \vec{c})$ = 4 Area of RS

Q.13 (A)

$(\vec{d} + \vec{a})\ .\ (\vec{a} \times (\vec{b} \times (\vec{c} \times \vec{d})))$

$(\vec{d} + \vec{a})\ .\ (\vec{a} + \{(\vec{b}.\vec{d})\vec{c} - (\vec{b}.\vec{c})\vec{d}\})$

$(\vec{d} + \vec{a})\ .\ [(\vec{b}.\vec{d})(\vec{a} \times \vec{c}) - (\vec{b}.\vec{c})(\vec{a} \times \vec{d})]$

$(\vec{b}.\vec{d})\ [\vec{a}\quad \vec{c}\quad \vec{d}]$

Q.14 (A)

$\underbrace{(\vec{a} \times \vec{b})}_{P} \times (\vec{r} \times \vec{c}) + \underbrace{(\vec{b} \times \vec{c})}_{W} \times (\vec{r} \times \vec{a}) + (\vec{c} \times \vec{a}) \times \underbrace{(\vec{r} \times \vec{b})}_{V}$

$= \vec{P} \times (\vec{r} \times \vec{c}) + \vec{w} \times (\vec{r} \times \vec{a}) + (\vec{c} \times \vec{a}) \times \vec{v}$

$= (\vec{P}.\vec{c})\vec{r} - (\vec{P}.\vec{r})\vec{c} + (\vec{W}.\vec{a})\vec{r} - (\vec{W}.\vec{r})\vec{a} + (\vec{c}.\vec{V})\vec{a} - (\vec{a}.\vec{V})\vec{c}$

$= [\vec{a}\ \vec{b}\ \vec{c}]\vec{r} - [\vec{a}\ \vec{b}\ \vec{r}]\vec{c} + [\vec{b}\ \vec{c}\ \vec{a}]\vec{r} - [\vec{b}\ \vec{c}\ \vec{r}]\vec{a} +$

$[\vec c\ \vec r\ \vec b]\vec a - [\vec a\ \vec r\ \vec b]\vec c$

$= 2[\vec a\ \vec b\ \vec c]\vec r$

Q.15 (D)

$\vec a = (1, 1, 1)$, $\vec b = (1, -1, 2)$,

$\vec c = (x, x-2, -1)$

$$\vec a \times \vec b = \begin{vmatrix} \hat i & \hat j & \hat k \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = (3, -1, -2)$$

$\vec c.(\vec a \times \vec b) = 0$

$3x - (x-2) + 2 = 0 \quad \Rightarrow \boxed{x = -2}$

Q.16 (D)

$\vec a = (x, y, 2)$ $\vec b = (1, -1, 1)$

$\vec c = (1, 2, 0)$

$\vec a.\vec b = 0$ $\vec a.\vec c = 4$

$x - y + 2 = 0$ ... (1)

$x + 2y = 4$ ...(2)

$x = 0, y = 2$

$\vec a = (0, 2, 2)$

$[\vec a\ \ \vec b\ \ \vec c] = \vec a\ .\ (\vec b \times \vec c)$

$\vec b \times \vec c = (-2, 1, 3)$ = (0, 2, 2) . (–2, 1, 3) = 2 + 6 = 8 | a $|^2$

Q.17 (C)

$\vec b \times \vec d = 0 \Rightarrow \vec b = \lambda \vec d$

$\vec a = \vec b + \vec c$ & $\vec c.\vec d = 0$

$\Rightarrow \vec a.\vec d = \vec b.\vec d + \vec c.\vec d$

or $\vec a.\vec d = \vec b.\vec d$

Now $\dfrac{\vec d \times (\vec a \times \vec d)}{\vec d^2} = \dfrac{(\vec d.\vec d)\vec a - (\vec d.\vec a)\vec d}{\vec d^2}$

$= \vec a - \dfrac{(\vec b.\vec d)\vec d}{\vec d^2} = \vec a - \dfrac{(\lambda\ \vec d.\vec d)\vec d}{\vec d^2} = \vec a - \lambda \vec d = \vec a - \vec b = \vec c$

Q.18 (C)

$(\vec a \times \vec b) \times \dfrac{(\vec b \times \vec c)}{\vec v}$

$= (\vec a \times \vec b) \times \vec v$

$= (\vec a\ .\ \vec v)\ \vec b - (\vec b \cdot \vec v)\ \vec a$

$= [\vec a\ \vec b\ \vec c]\ \vec b$

$(\vec b \times \vec c) \times (\vec c \times \vec a) = [\vec b\ \vec c\ \vec a]\ \vec c$

$(\vec c \times \vec a) \times (\vec a \times \vec b) = [\vec c\ \vec a\ \vec b]\ \vec a$

So box product = $[\vec a\ \vec b\ \vec c]^3 [\vec a\ \vec b\ \vec c]$

$= [\vec a\ \vec b\ \vec c]^4$

Q.19 (A)

= (a, 1, 1), = (1, b, 1), = (1, 1, c)

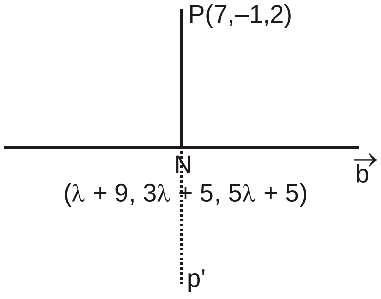
$$\left[\vec A\ \ \vec B\ \ \vec C\right] = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$= \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

Q.20 (B)

$\overline{PN} = (\lambda + 2, 3\lambda + 6, 5\lambda + 3)$

$\vec b = (1, 3, 5)$ $\overline{PN} \cdot \vec b = 0$



$(\lambda + 2) + 3(3\lambda + 6) + 5(5\lambda + 3) = 0$

$\Rightarrow 10(\lambda + 2) + 5(5\lambda + 3) = 0$

$\Rightarrow 10\lambda + 20 + 25\lambda + 15 = 0$

$\Rightarrow 35\lambda + 35 = 0 \Rightarrow \lambda = -1$

$\vec N = (8, 2, 0) \Rightarrow \vec N = \dfrac{\vec p + \vec p'}{2}$

$\Rightarrow$ p' = 2N – P = 2(8, 2, 0) – (7, –1, 2) = (16, 4, 0) – (7, –1, 2) = (9, 5, – 2)

Q.21 (C)

$\vec r.\vec n = 1;\ \vec r = \vec a + \vec b$

Direction of lie will be = $(\vec b \times a)$

passing through = $\vec{c}$

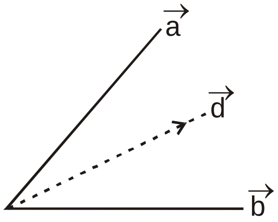
$$\boxed{\vec{r} = \vec{c} + \lambda(\vec{b} \times \vec{n})}$$

Q.22 (A)

$\vec{d} = \hat{a} + \hat{b}$

$$= \frac{-4\hat{i} + 3\hat{k}}{5} + \frac{|14\hat{i} + 2\hat{j} - 5\hat{k}|}{15}$$

$$= \frac{-12\hat{i} + 9\hat{k} + 14\hat{i} + 2\hat{j} - 5\hat{k}}{15}$$



$$= \frac{2\hat{i} + 2\hat{j} + 4\hat{k}}{15} = \frac{2}{15}(\hat{i} + \hat{j} + \hat{k})$$

Q.23 (A)

$\vec{r} = \vec{a} + \lambda\vec{p}$, $\vec{r} \,.\, \vec{n} = 14$ so $\vec{p} \,.\, \vec{n} = 0$

(2, 1, 12) . (3, –2, –m) = 0

$6 - 2 - 2m = 0 \Rightarrow m = 2$

Q.24 (C)

$\vec{r} \,.\, (\vec{a} + \vec{b} + \vec{c}) = 0$

$\vec{r} = l(\vec{b} \times \vec{c}) + m\,(\vec{c} \times \vec{a}) + n(\vec{a} \times \vec{b})$

$\vec{r}.\vec{a} = l[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ ...(i)

$\vec{r}.\vec{b} = m[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ ...(ii)

$\vec{r}.\vec{c} = n[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ ...(iii)

Add them $\vec{r} \,.\, (\vec{a} + \vec{b} + \vec{c}) = (l + m + n)\ [\vec{a}\ \ \vec{b}\ \ \vec{c}]$

$\Rightarrow l + m + n = 0$

Q.25 (D)

We have $V = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2\operatorname{cosec}\alpha & 1 \\ 0 & 1 & 2\operatorname{cosec}\alpha \end{vmatrix}$

$= 4\operatorname{cosec}^2\alpha - 1 - 2\operatorname{cosec}\alpha$

$$= 4\left[\operatorname{cosec}^2\alpha - \frac{1}{2}\operatorname{cosec}\alpha\right] - 1$$

$$= 4\left(\operatorname{cosec}\alpha - \frac{1}{4}\right)^2 - \frac{5}{4}$$

$$\therefore\ V_{least}\left(\alpha = \frac{\pi}{2}\right) = 4 \times \frac{9}{16} - \frac{5}{4} = \frac{4}{4} = 1.$$

NUMERCIAL VALUE BASED

Q.1 [2]

Angle between vector $\vec{a}$ & $\vec{b}$ remains same even if we presume them as unit vector. Here for sake of convinience let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are unit vectors.

$\vec{a}.\vec{b} = \cos\frac{\pi}{3} = \frac{1}{2}$ ....(1) ;

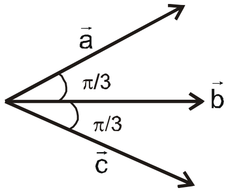
$\vec{b}.\vec{c} = \frac{1}{2}$ ....(2)

$\vec{a}.\vec{d} = \cos\alpha$ ....(3)

; $\vec{b}.\vec{d} = \cos\beta$ ....(4)

also $\vec{b} = \lambda\ (\vec{a} + \vec{c})$

Since $\vec{b}$ is presumed as unit vector



$\left|\lambda(\vec{a} + \vec{c})\right| = 1 \Rightarrow \lambda^2(\vec{a}^2 + \vec{c}^2 + 2\vec{a}.\vec{c}) = 1$

or $\lambda^2(1 + 1 - 1) = 1 \Rightarrow \lambda = 1$

$\therefore\ \vec{b} = (\vec{a} + \vec{c}) \Rightarrow \vec{c} = \vec{b} - \vec{a}$

again $\vec{d}.\vec{c} = \left|\vec{d}\right|\left|\vec{c}\right|\cos\theta = \vec{d}.(\vec{b} - \vec{a})$

$\Rightarrow \cos\theta = \cos\beta - \cos\alpha \Rightarrow \theta = \cos^{-1}(\cos\beta - \cos\alpha)$

Q.2 [9]

$(\vec{R} - \vec{C}) \times \vec{B} = \vec{O} \Rightarrow \vec{R} = \vec{C} + \lambda\vec{B}$

$\Rightarrow A.C + \lambda A.B = 0 \Rightarrow 15 + 3\lambda = 0$

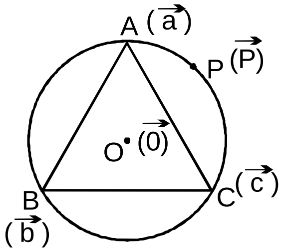
$\Rightarrow \lambda = -5 \Rightarrow \vec{R} = -\hat{i} - 8\hat{j} + 2\hat{k}$

Q.3 [6]

Let O $(\vec{0})$ be the circumcentre of ΔABC

Given $(\vec{a}) = |\vec{b}| = |\vec{c}| = R \Rightarrow 0 = \dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$

$\Rightarrow \vec{a}+\vec{b}+\vec{c} = 0$



$$\frac{|\overrightarrow{PA}|^2 + |\overrightarrow{PB}|^2 + |\overrightarrow{PC}|^2}{R^2} = ?$$

$$\Rightarrow \frac{|\vec{P}-\vec{a}|^2 + |\vec{P}-\vec{b}|^2 + |\vec{P}-\vec{c}|^2}{R^2}$$

$$\frac{3|\vec{P}| + |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 - 2\vec{P}.(\vec{a}+\vec{b}+\vec{c})}{R^2} \quad ;$$

$$\frac{6R^2}{R^2} = 6$$

Q.4 [72]

vectors $\vec{a}, \vec{b}$ & $\vec{c}$ are non coplanar so are the vectors

$\vec{a}\times\vec{b},\ \vec{b}\times\vec{c}$

Let position vector of circumcentre

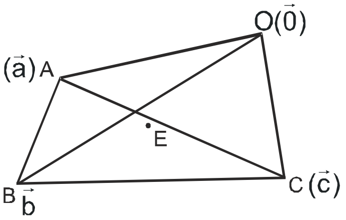
$\vec{r} \equiv x(\vec{a}\times\vec{b}) + y(\vec{b}\times c) + z(\vec{c}\times\vec{a})$

also OE = AE = EB = EC

$\Rightarrow |\vec{r}| = |\vec{r}-\vec{a}| = |\vec{r}-\vec{b}| = |\vec{r}-\vec{c}|$

or $\vec{r}^2 = \vec{r}^2 + \vec{a}^2 - 2\vec{r}.\vec{a} =$

$\vec{r}^2 + \vec{b}^2 - 2\vec{r}.\vec{b} = \vec{r}^2 + \vec{c}^2 - 2\vec{r}.\vec{c}^2$



$\Rightarrow 2\vec{r}.\vec{a} = \vec{a}^2$, $\quad 2\vec{r}.\vec{b} = \vec{b}^2$, $2\vec{r}.\vec{c} = \vec{c}^2$ or

$2y\,[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}^2 \quad \Rightarrow \quad y = \dfrac{\vec{a}^2}{2[\vec{a}\ \vec{b}\ \vec{c}]}$

Similarly z & x can be obtained

Q.5 [18]

Equation of line AB is $\vec{r} = \vec{a} + \lambda(\vec{b}-\vec{a})$

$\Rightarrow \vec{r} = (\hat{i}+2\hat{j}+\hat{k}) + \lambda(\hat{i}-\hat{j}+\hat{k}) \quad ....(1)$

$\overrightarrow{CD} = 2\hat{i}+2\hat{j}-2\hat{k}$;

$\overrightarrow{CE} = 4\hat{i}+5\hat{j}-2\hat{k}$; $\quad \vec{n} = \overrightarrow{CD}\times\overrightarrow{CE}$

$= 6\hat{i}-4\hat{j}+2\hat{k}$

So equation of plane CDE is 3x – 2y + z = 12. Solve with line $\vec{r} = (\hat{i}+2\hat{j}+\hat{k}) + \lambda(\hat{i}-\hat{j}+\hat{k})$

$3(1+\lambda) - 2(2-\lambda) + 1 + \lambda = 12$

$\Rightarrow \lambda = 2$ Hence R is $3\hat{i}+3\hat{k}$

Q.6 [36]

Equation of line L_1 is $7\hat{i}+6\hat{j}+2\hat{k}$ + λ $(-3\hat{i}+2\hat{j}+4\hat{k})$

Equation of line L_2 is $5\hat{i}+3\hat{j}+4\hat{k}$ + μ $(2\hat{i}+\hat{j}+3\hat{k})$

$\overrightarrow{CD} = 2\hat{i}+3\hat{j}-2\hat{k} + \lambda\ (-3\hat{i}+2\hat{j}+4\hat{k}) - \mu\ (2\hat{i}+\hat{j}+3\hat{k})$. since it is parallel to $2\hat{i}-2\hat{j}-\hat{k}$

$$\therefore \frac{2-3\lambda-2\mu}{2} = \frac{3+2\lambda-\mu}{-2} = \frac{-2+4\lambda-3\mu}{-1}$$

$\therefore \lambda = 2,\ \mu = 1$

$\therefore \overrightarrow{CD} = -6\hat{i}+6\hat{j}+3\hat{k} \qquad \therefore$

$|4\overrightarrow{CD}| = 36$

Q.7 [35]

We have $\vec{V}_1 \cdot \vec{V}_2 = 2(\sin\alpha + \cos\alpha)\sin\beta + \cos\beta$

$\therefore \quad \vec{V}_1 \cdot \vec{V}_2 = 3$

[given that 2(sin α + cos α)sin β + cos β – 3]

$\left|\vec{V}_1\right|^2 \left|\vec{V}_2\right|^2 \cos^2\theta = 9 \quad \left(\theta = \vec{V}_1 {}^\wedge \vec{V}_2\right)$

$[4(1 + \sin 2\alpha) + 1]\,(1)\cos^2\theta = 9$

$(5 + 4\sin 2\alpha)\cos^2\theta = 9$

$\cos^2\theta = \dfrac{9}{5+4\sin 2\alpha} \le 1 \Rightarrow 9 \le 5 + 4\sin 2\alpha$

$\Rightarrow 4 \le 4\sin 2\alpha \Rightarrow \sin 2\alpha \ge 1$

$\therefore \quad \sin 2\alpha = 1 \Rightarrow \alpha = \dfrac{\pi}{4}$

$\therefore \quad \cos^2\theta = \dfrac{9}{9} = 1 \Rightarrow \theta = 0$

Hence $\theta = 0 \Rightarrow \vec{V}_1$ and $\vec{V}_2$ are collinear i.e.

$$\frac{2(\sin\alpha+\cos\alpha)}{\sin\beta} = \frac{1}{\cos\beta} \qquad \left(\alpha = \frac{\pi}{4}\right)$$

$$\tan\beta = 2\left(\frac{2}{\sqrt{2}}\right) = 2\sqrt{2}$$

Hence $3\tan^2\alpha + 4\tan^2\beta = 3 + (4)(8) = 35$

Q.8 [5]

Let P and Q be (x_1, y_1) and (x_2, y_2)

$\therefore\ \overrightarrow{OP}\cdot\hat{i} = x_1 = 2$ and $\overline{OQ}\cdot\hat{i} = x_2 = -2$

Let $y = f(x) = x^7 - 2x^5 + 5x^3 + 8x + 5$

$\therefore\ y_1 = f(x_1) = f(2)$ and $y_2 = f(x_2) = f(-2)$

$\therefore\ \left|\overline{OP}+\overline{OQ}\right| = x_1\hat{i}+y_1\hat{j}+x_2\hat{i}+y_2\hat{j}$

$= \sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$

$= \sqrt{\left(f(2)+f(-2)\right)^2}$

$= \left(f(2)+f(-2)\right)\hat{j}$

So, magnitude of $\overrightarrow{OP}+\overrightarrow{OQ} = f(2) + f(-2) = 10$

(from the given functional rule)

$\Rightarrow\ 2M = 10\ \Rightarrow\ M = 5$ **Ans.**

Q.9 [0]

$\vec{a}+\vec{b}=\lambda\vec{c}$ ....(1)

$\vec{b}+\vec{c}=\mu\vec{a}$ ....(2)

$\vec{a}-\vec{c}=\lambda\vec{c}-\mu\vec{a}$

$\lambda = -1$, $\mu = -1\ \Rightarrow\ \vec{a}+\vec{b}=-\vec{c}$ from (1)

$\therefore\quad \vec{a}+\vec{b}+\vec{c}=0$. **Ans.**

Q.10 [34]

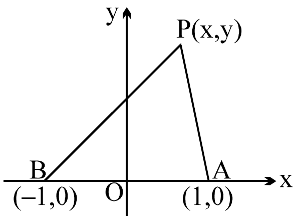
Let P be (x, y)

$\overrightarrow{PA} = (1-x)\hat{i}-y\hat{j}$; $\quad \overrightarrow{PB} = (-1-x)\hat{i}-y\hat{j}$

$\therefore\quad \left(\overrightarrow{PA}\cdot\overrightarrow{PB}\right) = \left((x-1)\hat{i}+y\hat{j}\right)\cdot\left((x+1)\hat{i}+y\hat{j}\right) = (x^2-1)+y^2$

also $\quad 3\left(\overrightarrow{OA}\cdot\overrightarrow{OB}\right) = 3\hat{i}\cdot(-\hat{i}) = -3$

hence $\quad \left(\overrightarrow{PA}\cdot\overrightarrow{PB}\right) + 3\left(\overrightarrow{OA}\cdot\overrightarrow{OB}\right) = 0$



$x^2 - 1 + y^2 - 3 = 0 = 0$

$x^2 + y^2 = 4$ ....(1)

which gives the locus of P i.e. P move on a circle with centre (0, 0) and radius 2.

now

$\left|\overrightarrow{PA}\right|^2 = (x-1)^2 + y^2$; $\quad \left|\overrightarrow{PB}\right|^2 = (x+1)^2 + y^2$

$\therefore\ \left|\overrightarrow{PA}\right|^2\left|\overrightarrow{PB}\right|^2 = (x^2 + y^2 - 2x + 1)(x^2 + y^2 + 2x + 1)$

$= (5 - 2x)(5 + 2x)$ [using $x^2 + y^2 = 4$]

$\therefore\ \left|\overrightarrow{PA}\right|^2\left|\overrightarrow{PB}\right|^2 = 25 - 4x^2$ subject to $x^2 + y^2 = 4$

$\left.\left|\overrightarrow{PA}\right|^2\left|\overrightarrow{PB}\right|^2\right|_{\text{min.}} = 25 - 16 = 9$; (when x = 2 or – 2)

and $\left.\left|\overrightarrow{PA}\right|^2\left|\overrightarrow{PB}\right|^2\right|_{\text{max.}} = 25 - 0 = 25$ (when x = 0)

$$3 \le \left|\overrightarrow{PA}\right|\left|\overrightarrow{PB}\right| \le 5$$

hence M = 5 and m = 3 ⇒ $M^2 + m^2 = 34$ Ans.

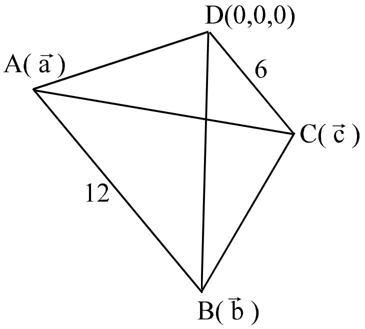
Q.11 [48]

Given $|\vec{b}-\vec{a}|=12$; $|\vec{c}| = 6$

Equation of CD is $\vec{r}=\lambda\vec{c}$

& eq. of AB is $\vec{r}=\vec{a}+\mu(\vec{b}-\vec{a})$

S.D. = $\left|\dfrac{\vec{a}\cdot\vec{c}\times(\vec{b}-\vec{a})}{\vec{c}\times(\vec{b}-\vec{a})}\right| = 8$



$\left|\dfrac{[\vec{a}\ \ \vec{c}\ \ \vec{b}]}{\vec{c}\times(\vec{b}-\vec{a})}\right| = 8$

$\left|[\vec{a}\ \ \vec{c}\ \ \vec{b}]\right| = 8\left|\vec{c}\times(\vec{b}-\vec{a})\right|$

6V = $\dfrac{8|\vec{c}||\vec{b}-\vec{a}|}{2}$

⇒ 6V = (4)(6)(12) ⇒ V = 48

Q.12 [5]

$(\vec{p}\cdot\vec{r})\vec{q}-(\vec{q}\cdot\vec{r})\vec{p}+(\vec{q}\cdot\vec{r})\vec{q}$

$= (x^2+y^2)\vec{q} + (14-4x-6y)\vec{p}$

$\therefore\ \vec{p}\cdot\vec{r}+\vec{q}\cdot\vec{r} = x^2 + y^2$ ...(1)

and $-(\vec{q}\cdot\vec{r}) = 14 - 4x - 6y$ ...(2)

From (1) + (2)

$\vec{p}\cdot\vec{r} = x^2 + y^2 - 4x - 6y + 14$ ...(3)

$\because\ (\vec{r}\cdot\vec{r})\vec{p} = \vec{r}$

Taking dot product with $\vec{r}$, we get

$(\vec{r}\cdot\vec{r})(\vec{p}\cdot\vec{r}) = \vec{r}\cdot\vec{r} \Rightarrow \vec{p}\cdot\vec{r} = 1$

$\therefore$ from(3)

$x^2 + y^2 - 4x - 6y + 14 = 1$

$\Rightarrow (x-2)^2 + (y-3)^2 = 0 \Rightarrow x = 2$ & $y = 3$

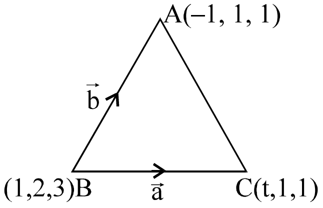
Hence $(x + y) = 5$. **Ans.**

Q.13 $[\frac{\sqrt{3}}{2}]$

$A = \frac{1}{2}\left|\vec{a}\times\vec{b}\right|$ and $\left|\vec{a}\times\vec{b}\right|^2 = \vec{a}^2\,\vec{b}^2 - \left(\vec{a}^2\cdot\vec{b}^2\right)^2$

$\vec{a} = (t-1)\hat{i} - \hat{j} - 2\hat{k}$; $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$

$|\vec{a}|^2 = (t-1)^2 + 1 + 4$; $|\vec{b}|^2 = 4 + 1 + 1 = 6$



$\vec{a}\cdot\vec{b} = 2(t-1) - 1 - 2 = 2t - 5$

$|\vec{a}\times\vec{b}|^2 = 6[t^2 - 2t + 6] - (4t^2 + 25 - 20t)$

$|\vec{a}\times\vec{b}|^2 = 2t^2 + 8t + 11$

which is minimum at t = – 2

$|\vec{a}\times\vec{b}|^2_{min} = 8 - 16 + 11 = 3$

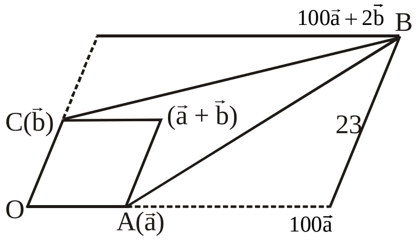
$|\vec{a}\times\vec{b}|_{min} = \sqrt{3}$

$\therefore\ \frac{|\vec{a}\times\vec{b}|_{min}}{2} = A_{min} = \frac{\sqrt{3}}{2}$ Ans.

Q.14 [51]

$P = \left|\left(\overline{OA}\right)\times\left(\overline{OC}\right)\right|$

$P = \left|\vec{a}\times\vec{b}\right|$



$Q = \frac{1}{2}\left|\overline{OB}\times\overline{AC}\right| = \frac{1}{2}\left|\left(100\vec{a}+2\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|$

$= \frac{1}{2}\left|-100\vec{a}\times\vec{b}+2\vec{b}\times\vec{a}\right|$

$= \frac{1}{2}\left|102\left(\vec{a}\times\vec{b}\right)\right| = 51\left|\vec{a}\times\vec{b}\right|$

Now $Q = \lambda P$

$51\left|\vec{a}\times\vec{b}\right| = \lambda\left|\vec{a}\times\vec{b}\right|$

$\lambda = 51$ **Ans.**

KVPY

PREVIOUS YEAR'S

Q.1 (D)

$\vec{u}\times\vec{v} = \left(2\hat{i}-\hat{j}+\hat{k}\right)\times\left(-3\hat{j}+2\hat{k}\right)$

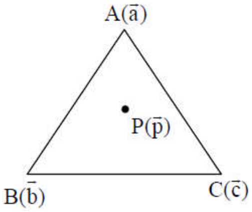
$-6\hat{k}-4\hat{j}-2\hat{i}+3\hat{i} = \hat{i}-4\hat{j}-6\hat{k}$

Let $\vec{w} = a\hat{i}+b\hat{j}$ $a^2+b^2=1$

$a = \cos\theta$; $b = \sin\theta$

$\max.\text{value} = \sqrt{1^2+(-4)^2} = \sqrt{17}$

Q.2 (D)



$\overline{PA}+2\overline{PB}+3\overline{PC} = \vec{0}.$

$(\vec{a}-\vec{p})+2\left(\vec{b}-\vec{p}\right)+3(\vec{c}-\vec{p}) = 0$

$$\vec{P} = \frac{\vec{a}+2\vec{b}+3\vec{c}}{6}$$

$$\frac{\text{Area}\,\Delta\,ABC}{\text{Area}\,\Delta\,APC} = \frac{\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}{\frac{1}{2}|\vec{a}\times\vec{p}+\vec{p}\times\vec{c}\times\vec{c}\times\vec{a}|}$$

Put $\vec{P} = \frac{\vec{a}+2\vec{b}+3\vec{c}}{6}$

ratio = 3

Q.3 (A)

$\vec{a}\times\vec{b}+\vec{c}\times\vec{b}=0$ similarly $\vec{b}+\vec{c}=\lambda_2\vec{a}$

$\vec{a}+\vec{c}=\lambda_1\vec{b}$ $\qquad$ $\vec{b}+\vec{a}=\lambda_3\vec{c}$

Hence $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Downarrow$

Only 1 position of centroid

Q.4 (C)

G is centroid

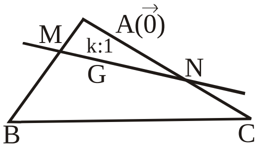
$$G = \frac{A+B+C}{3}$$

$$G = \frac{20+H}{3}$$

2O + H = 3G

$$\begin{aligned}\overrightarrow{HA}+\overrightarrow{HB}+\overrightarrow{HC} &= \vec{A}-\vec{H}+\vec{B}-\vec{H}+\vec{C}-\vec{H}\\ &= \vec{A}+\vec{B}+\vec{C}-3\vec{H}\\ &= 3\vec{G}-3\vec{H}\\ &= 2\vec{O}+\vec{H}-3\vec{H}\\ &= 2\vec{O}-2\vec{H}\\ &= 2\overrightarrow{HO}\end{aligned}$$

Q.5 (C)



Let $\overrightarrow{AB}=\vec{b}, \overrightarrow{AC}=\vec{c}$

$\overrightarrow{AM}=\lambda\vec{b}$

$\overrightarrow{AN}=m\vec{c}$

Let G divides MN in the ratio k : 1

So $\frac{k\mu\,\vec{c}+\lambda\vec{b}}{k+1} = \frac{\vec{b}+\vec{c}}{3}$

$\Rightarrow \frac{k\mu}{k+1}=\frac{1}{3}$ $\qquad$ $\frac{\lambda}{k+1}=\frac{1}{3}$

$\Rightarrow k = \frac{\lambda}{\mu}$

$\Rightarrow \frac{1}{\lambda}+\frac{1}{\mu}=3$

AM $\geq$ GM

$$\frac{\frac{1}{\lambda}+\frac{1}{\mu}}{2} \geq \frac{1}{\sqrt{\lambda\mu}} \Rightarrow \left(\frac{2}{3}\right)^2 \leq \lambda\mu \quad ....(1)$$

Now, $\frac{\text{area of }\Delta AMN}{\text{area of }ABC} = \frac{\frac{1}{2}\lambda\mu\left|\vec{b}\times\vec{c}\right|}{\frac{1}{2}\left|\vec{b}\times\vec{c}\right|} = \lambda\mu$

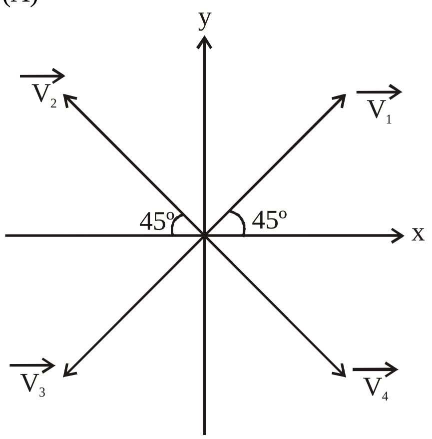
using $\frac{1}{\lambda}+\frac{1}{\mu}=3 \Rightarrow \text{Ratio} = \frac{\lambda}{3\lambda-1}\lambda \in [0,1]$ maximum

value of ratio $= \frac{\lambda^2}{3\lambda-1}$ attain when λ=1using derivative

but l is not 1 because M is an interior point.

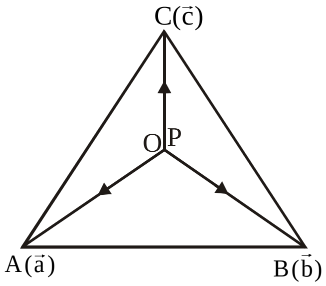
So $\frac{4}{9} \leq \text{ratio} < \frac{1}{2}$

Q.6 (A)



In this case B, C, D are not possible.

Q.7 (C)



$\vec{a},\vec{b},\vec{c}$ are unit vectors

$$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \frac{\pi}{3}$$

$$\text{centre } p\left(\frac{\vec{0}+\vec{a}+\vec{b}+\vec{c}}{4}\right)$$

Now angle between $\overrightarrow{AP}$ & $\overrightarrow{BP}$

$$\cos\theta = \frac{\overrightarrow{AP}.\overrightarrow{BP}}{|\overrightarrow{AP}||\overrightarrow{BP}|} = \frac{\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4}-\vec{a}\right).\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4}-\vec{b}\right)}{\left|\frac{\vec{a}+\vec{b}+\vec{c}}{\vec{4}}-\vec{a}\right|\left|\frac{\vec{a}+\vec{b}+\vec{c}}{4}-\vec{b}\right|}$$

$$= \frac{\left(\vec{b}+\vec{c}-3\vec{a}\right).\left(\vec{a}+\vec{c}-3\vec{b}\right)}{|\vec{b}+\vec{c}-3\vec{a}|.|\vec{a}+\vec{c}-3\vec{b}|}$$

$$= \frac{\vec{a}.\vec{b}+\vec{b}.\vec{c}-3\vec{b}^2+\vec{a}.\vec{c}+\vec{c}^2-3\vec{b}\,.\,\vec{c}-3\vec{a}^2-3\vec{a}.\vec{c}+9\vec{a}\,.\vec{b}}{(\vec{b}^2+\vec{c}^2+9\vec{a}^2+2\vec{b}\,.\,\vec{c}-6\vec{a}\,.\,\vec{c}-6\vec{a}.\vec{b})}$$

$$= \frac{\frac{1}{2}+\frac{1}{2}-3+\frac{1}{2}+1-\frac{3}{2}-3-\frac{3}{2}+\frac{9}{2}}{1+1+9+1-3-3}$$

$$= \frac{-5+3}{6} = -\frac{1}{3}$$

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

Q.8 (B)

$$\vec{b} = \lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\vec{a} = \vec{b}+\vec{c}$$

$$\vec{c} = \vec{a}-\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\vec{c} = \left(6\hat{i}-3\hat{j}-6\hat{k}\right)-\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\vec{c} = \left(6-\lambda\right)\hat{i}+\left(-3-\lambda\right)\hat{j}+\left(-6-\lambda\right)\hat{k}$$

$$\vec{c}\cdot\vec{\lambda} = 6-\lambda-3-\lambda-6-\lambda = 0$$

$3\lambda = 3$

$\lambda = -1$

$$\vec{c} = 7\hat{i}-2\hat{j}-5\hat{k}$$

Q.9 (C)

V is the circumcentre of ΔABC

$\forall A \equiv (1,0),\ B \equiv (0,1)\ C(2,0)$

Let V (x,y)

VA = VB = VC

$(x-1)^2 + y^2 = x^2 + (y-1)^2 = (x-2)^2 + y^2$

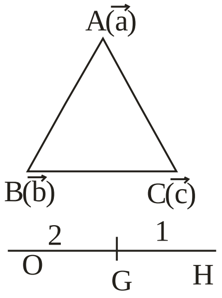
$$(x,y) = \left(\frac{3}{2},\frac{3}{2}\right)$$

$$V = \frac{3i+3j}{2}$$

$$|v| = \frac{3}{\sqrt{2}} \in (2,3)$$

Q.10 (C)

Circumcenter (Origin O)



$$(\vec{O}) = \left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right), H = \left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right),$$

$$\vec{N} = \frac{1}{4}(\vec{a}+\vec{b}+\vec{c})$$

Q.11 (C)

Equations

x + y + z = 0

ax + by + cz = 0,

$a^2x + b^2y + cz = 0$,

have a unique solution

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \neq 0 \quad \Rightarrow \ (a-b)(b-c)(c-a) \neq 0$$

Q.12 (A)

$|\vec{r}-\vec{b}|+|\vec{r}-\vec{c}|=4$ is an ellipsoid with foci $\vec{b},\vec{c}$.

When it is cut by plane $\vec{r}.\vec{a}$ = 5 i.e. x + y + z = 5 then we get ellipse with 2a = 4 and 2ae = $|\vec{b}-\vec{c}| = \sqrt{14}$.

Area of ellipse

$$= \pi ab = 2\pi\sqrt{a^2-a^2e^2} = 2\pi\sqrt{4-\frac{14}{4}} = \pi\sqrt{2}$$

Q.13 (B)

$$\left(\frac{a_1}{2}+\frac{a_2}{4}+\frac{a_3}{8}\right)^2 = \frac{a_1^2}{2}+\frac{a_2^2}{4}+\frac{a_3^2}{8}$$

$$16a_1{}^2 + 12a_2{}^2 + 7a_3{}^2 - 16a_1a_2 - 4a_2a_3 - 8a_1a_3 = 0$$

$$\Rightarrow \left(2\sqrt{2}a_1 - 2\sqrt{2}a_2\right)^2 + \left(2\sqrt{2}a_1 - 2\sqrt{2}a_3\right)^2$$

$$+ (2a_2 - a_3)^2 + 4a_3^2 = 0$$

$$\Rightarrow a_1 = a_2 = a_3 = 0$$

JEE-MAIN
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Q.1 (2)

$$\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$$

$$(\vec{r} - \vec{c}) \times \vec{a} = \vec{0}$$

$$\Rightarrow \vec{r} - \vec{c} = \lambda\vec{a}$$

$$\vec{r} = \lambda\vec{a} + \vec{c}$$

$$\vec{r} \cdot \vec{b} \;\; 0 \quad \Rightarrow (\vec{a} \cdot \vec{b}) + (\vec{b} \cdot \vec{c}) = 0$$

$$2\lambda - 1 = 0 \quad \Rightarrow \lambda = \frac{1}{2}$$

$$\therefore \quad \vec{r} = \frac{1}{2}\vec{a} + \vec{c}$$

$$\vec{r} = \frac{1}{2}(\hat{i} + \hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} - \hat{k})$$

$$\vec{r} = \frac{3}{2}\hat{i} - \frac{3}{2}\hat{j} - \frac{3}{2}\hat{k}$$

$$\vec{r} = \frac{3}{2}(\hat{i} - \hat{j} - \hat{k})$$

$$\vec{r} \cdot \vec{a} = \frac{3}{2}(\hat{i} - \hat{j} - \hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) = \frac{3}{2}(1–1+1) = \frac{3}{2}$$

Q.2 [2]

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \alpha & 1 \\ 1 & -\alpha & 3 \end{vmatrix} = 4\alpha\,\hat{i} - 8\,\hat{j} - 4\alpha\,\hat{k}$$

$$\text{area} = |\bar{a} \times \bar{b}| = 8\sqrt{3}$$

$$= \sqrt{16\alpha^2 + 16\alpha^2 + 64} = 8\sqrt{3}$$

$$= 32\alpha^2 + 64 = 64.3$$

$$\alpha^2 + 2 = 2.3 = 6 \Rightarrow a^2 = 4$$

$$\alpha = \pm 2$$

$$\bar{a} \cdot \bar{b} = 3 - \alpha^2 + 3 = 6 - 4 = 2$$

Q.3 (1)

$$\vec{a} \times (\vec{a} \times ((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2\, \vec{b}))$$

$$= \vec{a} \times (-|\vec{a}|^2(\vec{a} \times \vec{b})) = - -|\vec{a}|^2\, ((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2\, \vec{b})$$

$$= -(\vec{a} \cdot \vec{b})\vec{a}$$

$$= |\vec{a}|^4\, \vec{b} \qquad (\because\; \vec{a} \cdot \vec{b} = 0)$$

Q.4 (2)

$$\text{for points to be coplanar} \begin{vmatrix} 6 & 0 & -33 \\ 0 & y-5 & -28 \\ 2\lambda - 1 & -4 & -38 \end{vmatrix} = 0$$

$$\Rightarrow 6\,(-33\lambda + 165 - 112) + 33\,(2\lambda^2 - 11\lambda + 5) = 0$$

$$\Rightarrow -198\lambda + 318 + 66\lambda^2 - 363\lambda + 165 = 0$$

$$\Rightarrow 66\lambda^2 - 561\lambda + 483 = 0$$

$$\text{Sum} = \frac{561}{66} = \frac{187}{22} = \frac{17}{2}$$

Q.5 (1)

$$\frac{x}{1} = -\frac{1}{y} = \frac{1}{z} = \lambda(\text{let})$$

Unit vector parallel to

$$x\hat{i} + y\hat{j} + z\hat{k} = \pm\frac{\left(\lambda\hat{i} - \frac{1}{\lambda}\hat{j} + \frac{1}{\lambda}\hat{k}\right)}{\sqrt{\lambda^2 + \frac{2}{\lambda^2}}}$$

for λ=1 it is $\pm\frac{\left(\hat{i} - \hat{j} + \hat{k}\right)}{\sqrt{3}}$

Q.6 [75.00]

$$\vec{c} = \lambda\left(\vec{b} \times (\vec{a} \times \vec{b})\right)$$

$$= \lambda\left(\left(\vec{b}.\vec{b}\right) b - \left(\vec{b}.\vec{a}\right)\vec{b}\right)$$

$$= \lambda\left(5\left(-\hat{i} + \hat{j} + \hat{k}\right) + 2\hat{i} + \hat{k}\right)$$

$$= \lambda\left(-3\hat{i} + 5\hat{j} + 6\hat{k}\right)$$

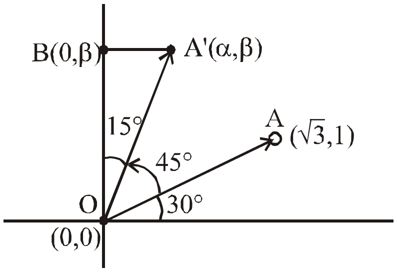
$$\vec{c}\;\vec{a} = 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$$

$$\lambda = \frac{1}{2}$$

$$\therefore\; 2\left|\left(\frac{-3}{2} - 1 + 2\right)\hat{i} + \left(\frac{5}{2} + 1\right)\hat{j} + (3 + 1 + 1)\hat{k}\right|^2$$

$$= 2\left(\frac{1}{4} + \frac{49}{4} + 25\right) = 25 + 50 = 75$$

Q.7 (1)



$$\text{Area of } \Delta\ (OA'B) = \frac{1}{2} OA'\cos 15° \times OA'\sin 15°$$

$$= \frac{1}{2}\left(OA'\right)^2 \frac{\sin 30°}{2}$$

$$= \left(3+1\right) \times \frac{1}{8} = \frac{1}{2}$$

Q.8 [28]

$$\vec{c} = \lambda(\vec{a} \times \vec{b})$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$(\vec{a} \times \vec{b}) = 3\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = \lambda(3\hat{i} - 2\hat{j} + \hat{k}) . (\hat{i} + \hat{j} + 3\hat{k})$$

$\Rightarrow \lambda(4) = 8 \Rightarrow \lambda = 2$

$$\vec{c} = 2(\vec{a} \times \vec{b})$$

$$\vec{c}.(\vec{a} \times \vec{b}) = 2\,|\vec{a} \times \vec{b}|^2 = 28$$

Q.9 (1)

$\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$

$\Rightarrow \vec{r} \quad (\vec{a} \quad \vec{b}) \quad 0$

$\Rightarrow \vec{r} = \lambda(\vec{a} - \vec{b})$

$\Rightarrow \vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$

Also $\vec{r}.(\hat{i} + 2\hat{j} + \hat{k}) = -3$

$\Rightarrow \lambda(-5 - 8 + 10) = -3$

$\lambda = 1$

Now $\vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$

$= \vec{r}.(2\hat{i} - 3\hat{j} + \hat{k})$

$= -10 + 12 + 10 = 12$

Q.10 [2]

$\vec{a}.\vec{b} = 1 \Rightarrow -\alpha\beta - \alpha\beta - 3 = 1$

$\Rightarrow -2\alpha\beta = 4 \Rightarrow \boxed{\alpha\beta = -2}$ ....(1)

$\vec{b}.\vec{c} = -3 \Rightarrow -\beta + 2\alpha + 1 = -3$

$\boxed{\beta - 2\alpha = 4}$ ....(2)

Solving (1) & (2), (α,β) = (–1, 2)

$$\frac{1}{3}[\vec{a}\ \vec{b}\ \vec{c}] = \frac{1}{3}\begin{vmatrix} \alpha & \beta & 3 \\ -\beta & -\alpha & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \frac{1}{3}\begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \frac{1}{3}\begin{vmatrix} 0 & 0 & 2 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix} = \frac{1}{3}[2(4-1)] = 2$$

Q.11 (2)

$\overrightarrow{OP} \perp \overrightarrow{OQ}$

$\Rightarrow -x + 2y - 3x = 0$

$\Rightarrow y = 2x$ .....(i)

$\left|\overrightarrow{PQ}\right|^2 = 20$

$\Rightarrow (x + 1)^2 + (y - 2)^2 + (1 + 3x)^2 = 20$

$\Rightarrow x = 1$

$\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}$ are coplanar.

$$\Rightarrow \begin{vmatrix} x & y & -1 \\ -1 & 2 & 3x \\ 3 & z & -7 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix} = 0$$

$\Rightarrow 1(-14 - 3z) - 2(7 - 9) - 1\,(-z - 6) = 0$

$\Rightarrow z = -2$

$\therefore x^2 + y^2 + z^2 = 1 + 4 + 4 = 9$

Q.12 [486]

Let $\vec{x} = \lambda\vec{a} + \mu\vec{b}$ (λ and μ are scalars)

$\vec{x} = \hat{i}(2\lambda + \mu) + \hat{j}(2\mu - \lambda) + \hat{k}(\lambda - \mu)$

Since $\vec{x} \cdot (3\hat{i} + 2\hat{j} - \hat{k}) = 0$

$3\lambda + 8\mu = 0$ .....(1)

Also Projection of $\vec{x}$ on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$

$$\frac{\vec{x} \cdot \vec{a}}{|\vec{a}|} = \frac{17\sqrt{6}}{2}$$

$6\lambda - \mu = 51$ .....(2)

From (1) and (2)

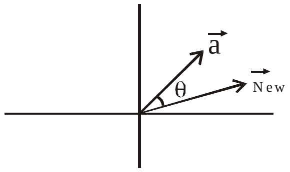
$\lambda = 8,\ \mu = -3$

$\vec{x} = 13\hat{i} - 14\hat{j} + 11\hat{k}$

$|\vec{x}|^2 = 486$

Q.13 (4)

$\vec{a}_{Old} = 3p\hat{i} + \hat{j}$



$\vec{a}_{New} = (p+1)\hat{i} + a\sqrt{10}\hat{j}$

$\Rightarrow |\vec{a}_{Old}| = |\vec{a}_{New}|$

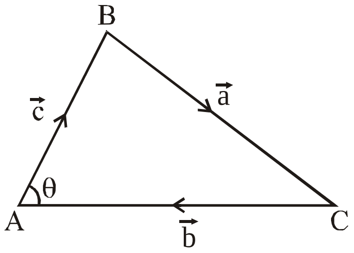
$\Rightarrow ap^2 + 1 = p^2 + 2p + 1 + 10$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)\ (p + 1) = 0 \rightarrow p = \frac{5}{4} - 1$

Q.14 (2)



$|\vec{a}| = 8, |\vec{b}| = 7, |\vec{c}| = 10$

$\cos\theta = \frac{|\vec{b}|^2 + |\vec{c}|^2 - |\vec{a}|^2}{2|\vec{b}||\vec{c}|} \quad \frac{17}{28}$

Projection of $\vec{c}$ on $\vec{b}$

$= |\vec{c}|\cos\theta$

$= 10 \quad \frac{17}{28}$

$= \frac{85}{14}$

Q.15 (2)

$|\vec{a}| = |\vec{b}|,\ |\vec{a} \times \vec{b}| = |\vec{a}|,\ \vec{a} \perp \vec{b}$

$|\vec{a} \times \vec{b}| = |\vec{a}| \Rightarrow |\vec{a}||\vec{b}|\sin 90° = |\vec{a}| \Rightarrow |\vec{b}| = 1 = |\vec{a}|$

$\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors.

Let $\vec{a} = \hat{i},\ \vec{b} = \hat{j} \Rightarrow \vec{a} \times \vec{b} = \hat{k}$

$\cos\theta = \frac{(\hat{i} + \hat{j} + \hat{k}).\hat{i}}{\sqrt{3}\ \sqrt{1}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Q.16 (2)

P(3, -1, 2)

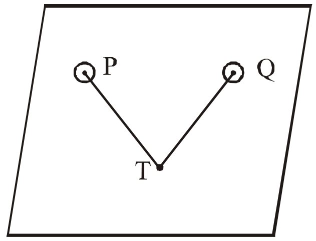
Q(1, 2, -4)

$\overrightarrow{PR} \parallel 4\hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{QS} \parallel -2\hat{i} + \hat{j} - 2\hat{k}$

dr's of normal to the plane containing P, T & Q will be proportional to :

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$$



$\therefore \frac{\ell}{0} = \frac{m}{4} = \frac{n}{2}$

For point, T : $\overrightarrow{PT} = \frac{x-3}{4} = \frac{y+1}{-1} = \frac{z-2}{2} = \lambda$

$\overrightarrow{QT} = \frac{x-1}{-2} = \frac{y-1}{1} = \frac{z+4}{-2} = \mu$

T : $(4\lambda + 3, -\lambda - 1, 2\lambda + 2)$

$\cong (2\mu + 1, \mu + 2, -2\mu - 4)$

$4\lambda + 3 = -2\mu + 1 \Rightarrow 2\lambda + \mu = 1$

$\lambda - \mu = -3 \Rightarrow \lambda = 2$

& $\mu = -5 \quad \lambda + \mu = -3 \quad \Rightarrow \lambda = 2$

So point T : (11, –3, 6)

$\overrightarrow{OA} = (11\hat{i} - 3\hat{j} + 6\hat{k}) \pm \left(\frac{2\hat{j} + \hat{k}}{\sqrt{5}}\right)\sqrt{5}$

$\overrightarrow{OA} = (11\hat{i} - 3\hat{j} + 6\hat{k}) \pm (2\hat{j} + \hat{k})$

$\overrightarrow{OA} = 11\hat{i} - \hat{j} + 7\hat{k}$

or

$9\hat{i} - 5\hat{j} + 5\hat{k}$

$|\overrightarrow{OA}| = \sqrt{121 + 1 + 49} = \sqrt{171}$

or

$\sqrt{81 + 25 + 25} = \sqrt{131}$

Q.17	(1)
Q.18	(4)
Q.19	(1)
Q.20	[3]
Q.21	(4)
Q.22	(1)
Q.23	[60]
Q.24	(2)
Q.25	[2]
Q.26	(2)
Q.27	[9]
Q.28	(4)
Q.29	[4]
Q.30	[81]
Q.31	(3)
Q.32	[6]
Q.33	[1494]
Q.34	(3)
Q.35	(3)
Q.36	(4)
Q.37	[5]
Q.38	(1)
Q.39	[56]
Q.40	[90]

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 [9]

$(\vec{r}-\vec{c})\times\vec{b} = 0$

$\vec{r} - \vec{c} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{c} + \lambda\vec{b} \quad \lambda \in R$

$\because \vec{r} . \vec{a} = 0$

$\Rightarrow (\vec{c} + \lambda\vec{b}) . \vec{a} = 0$

$\Rightarrow ((\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j})) .(-\hat{i} - \hat{k}) = 0$

$\Rightarrow ((1-\lambda)\hat{i} + (2+\lambda)\hat{j} + 3\hat{k}) . (-\hat{i} - \hat{k}) = 0$

$\Rightarrow \lambda - 1 - 3 = 0$

$\Rightarrow \lambda = 4$

so $\vec{r} . \vec{b} = (-3\hat{i} + 6\hat{j} + 3\hat{k}) . (-\hat{i} + \hat{j})$

$= 3 + 6 = 9$

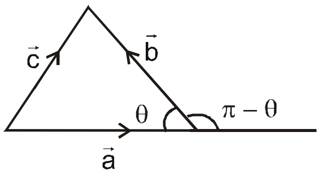
Q.2 **(A) → (q), (B) → (p), (C) → (s), (D) → (t)**

(A) $\cos(\pi-\theta) = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{-1+3}{\sqrt{1+3}\sqrt{1+3}} = \dfrac{2}{4}$

$-\cos\theta = \dfrac{1}{2}$

$\theta = \dfrac{2\pi}{3}$

(B) Using Leibeintz Theorem



f(b) – 3b = – 2b

f(b) = b

(C) $\dfrac{\pi^2}{\ell n3}\displaystyle\int_{7/6}^{5/6}(\sec\pi x)\,dx$

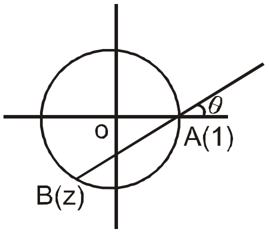
$$= \frac{\pi^2}{\ell n3}\left\{\frac{\ell n|\sec\pi x + \tan\pi x|}{\pi}\right\}_{7/6}^{5/6}$$

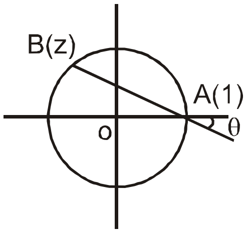
$$= \frac{\pi^2}{\ell n3}\left\{\frac{\ell n\left|\left(-\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)\right| - \ell n\left|-\frac{2}{3} + \frac{1}{\sqrt{3}}\right|}{\pi}\right\}$$

$$= \frac{\pi^2}{\ell n3}\left\{\frac{\ell n\left|\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{1}\right|}{\pi}\right\} = \pi$$

(D) z (z ≠ 1) lies on circle with center 0, radius 1

Arg $\left(\dfrac{1}{1-z}\right)$ = Arg 1 – Arg (1 – z) = angle between OA and BA

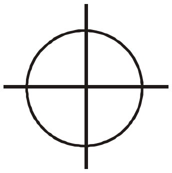


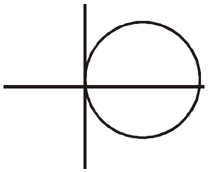


Absolute value of angle between OA and BA tends to $\frac{\pi}{2}$ as B tends to A.

Alter # 1

$$\left|\arg\left(\frac{1}{1-z}\right)\right| = |\arg 1 - \arg(1-z)| = |\arg(1-z)|$$





as |z| = 1 i.e. z lies on circle

$\Rightarrow$ – z lies on circle

$\Rightarrow$ 1 – z lies on circle

$\Rightarrow$ max |arg (1 – z)| = $\frac{\pi}{2}$

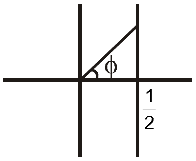
Alter # 2

$z = e^{i\theta}$

$$\frac{1}{1-z} = \frac{1}{2\sin^2\frac{\theta}{2} - i\sin\theta} = \frac{1}{2\sin\frac{\theta}{2}}$$

$$\left(\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}\right) = \frac{1}{2} + i\,\frac{1}{2}\cot\frac{\theta}{2}$$

Locus is $\frac{1}{1-z}$ is $x = \frac{1}{2}$



Maximum value of ϕ tends to $\frac{\pi}{2}$

Q.3 [3]

$$6 - 2\vec{a}\cdot\vec{b} - 2\vec{b}\cdot\vec{c} - 2\vec{c}\cdot\vec{a} = 9$$

$$\left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right) = \frac{-3}{2}$$

$$\left|\vec{a} + \vec{b} + \vec{c}\right|^2 \geq 0$$

$$3 + 2\left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right) \geq 0$$

$$\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} \geq \frac{-3}{2}$$

Since $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = \frac{-3}{2}$

$\Rightarrow \left|\vec{a} + \vec{b} + \vec{c}\right| = 0 \Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \left|2\vec{a} + 5(-\vec{a})\right| = |3\vec{a}| \Rightarrow 3$

Q.4 (C)

Let $\vec{c} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$

$\Rightarrow (\vec{a} + \vec{b}) \times \vec{c} = \vec{0}$

$\Rightarrow (\vec{a} + \vec{b}) \parallel \vec{c}$

Let $(\vec{a} + \vec{b}) = \lambda\vec{c}$

$\Rightarrow |\vec{a} + \vec{b}| = |\lambda|\,|\vec{c}|$

$\Rightarrow \sqrt{29} = |\lambda|.\ \sqrt{29}$

$\Rightarrow \lambda = \pm 1$

$\therefore\ \vec{a} + \vec{b} = \pm\ (2\hat{i} + 3\hat{j} + 4\hat{k})$

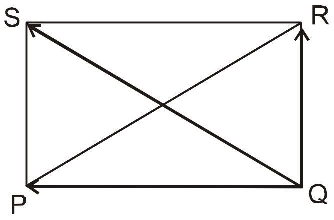
Now $(\vec{a} + \vec{b}).(-7\hat{i} + 2\hat{j} + 3\hat{k}) = \pm\,(-14 + 6 + 12) = \pm\, 4$

Q.5 (C)

$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{PS}$

$\overrightarrow{SQ} = \overrightarrow{PQ} - \overrightarrow{PS}$

$$\overrightarrow{PS} = \frac{\overrightarrow{PR} - \overrightarrow{SQ}}{2}$$



$$\overrightarrow{PQ} = \frac{\overrightarrow{PR} + \overrightarrow{SQ}}{2}$$

$$V = \left|\left[\overrightarrow{PQ}\ \overrightarrow{PS}\ \overrightarrow{PT}\right]\right|$$

$$V = \frac{1}{4}\left|\left[\overrightarrow{PR}+\overrightarrow{SQ}, \overrightarrow{PR}-\overrightarrow{SQ}, \overrightarrow{PT}\right]\right|$$

$$V = \frac{1}{2}\left|\left[\overrightarrow{PR}, \overrightarrow{SQ}, \overrightarrow{PT}\right]\right|$$

$$\frac{1}{2}\begin{vmatrix} 3 & 1 & -2 \\ 1 & -3 & -4 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\frac{1}{2}(-3-7-10) = 10$$

Q.6 ${}^{8}C_{3} - 24 = [32]$

Among set of eight vectors four vectors form body diagonals of a cube, remaining four will be parallel (unlike) vectors.

Numbers of ways of selecting three vectors will be ${}^{4}C_{3} \times 2 \times 2 \times 2 = 2^{5}$

Hence p = 5

Alternative

Eight vectors

$\vec{x} \equiv \hat{i}+\hat{j}+\hat{k}$

$\vec{y} \equiv \hat{i}+\hat{j}-\hat{k}$

$\vec{z} \equiv \hat{i}-\hat{j}+\hat{k}$

$\vec{\omega} = \hat{i}-\hat{j}-\hat{k}$

$\vec{x}' = -\hat{i}-\hat{j}-\hat{k}$

$\vec{y}' = -\hat{i}-\hat{j}+\hat{k}$

$\vec{z}' = -\hat{i}+\hat{j}-\hat{k}$

$\vec{\omega}' = -\hat{i}+\hat{j}+\hat{k}$

If we take $\vec{x}, \vec{x}'$ and any one of remaining sin x, vectors will always be coplaner

$\therefore$ No. of coplaner vectors = 6

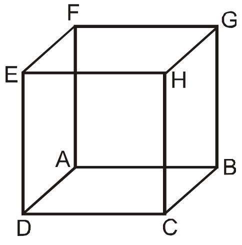
similarly on taking $\vec{y}, \vec{y}'$ = 6

$z, \vec{z}'$ = 6

$\omega, \vec{\omega}'$ = 6 $\therefore$ No. of set of coplaner vectors = 24

Alternative

A(0, 0, 0)
B(1, 0, 0)
C(1, 0, 1)
D(0, 0, 1)
E(0, 1, 1)
F(0, 1, 0)
G(1, 1, 0)
H(1, 1, 1)



$\overrightarrow{AH} = \hat{i}+\hat{j}+\hat{k}$

$\overrightarrow{BE} = -\hat{i}+\hat{j}+\hat{k}$

$\overrightarrow{CF} = -\hat{i}+\hat{j}-\hat{k}$

$\overrightarrow{DG} = \hat{i}+\hat{j}-\hat{k}$

Non-coplaner

Q.7 (C)

(P) $[\vec{a}\vec{b}\vec{c}] = 2$

$2\left(\vec{a}\times\vec{b}\right), 3\left(\vec{b}\times\vec{c}\right), \left(\vec{c}\times\vec{a}\right)$

$6\,[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = 6\,[\vec{a}\ \vec{b}\ \vec{c}]^{2}$

$= 6 \times 4 = 24$

$P \rightarrow 3$

(Q) $[\vec{a}\vec{b}\vec{c}] = 5$

$[3(\vec{a}+\vec{b})\ \ (\vec{b}+\vec{c})\ \ 2(\vec{c}+\vec{a})\,]$

$= 6 \times 2\,[\vec{a}\vec{b}\vec{c}]$

$= 12 \times 5 = 60$

$Q \rightarrow 4$

(R) $\frac{1}{2}|\vec{a}\times\vec{b}| = 20$

$$\Delta_1 = \frac{1}{2}\,|(2\vec{a}+3\vec{b}) \times (\vec{a}-\vec{b})|$$

$$= \frac{1}{2}\,|-2\vec{a}\times\vec{b} - 3(\vec{a}\times\vec{b})|$$

$$= \frac{5}{2}|\vec{a}\times\vec{b}|$$

$= 5 \times 20 = 100$

$R \rightarrow 1$

(S) $|\vec{a}\times\vec{b}| = 30$

$|(\vec{a}+\vec{b}) \times \vec{a}| = |\vec{b}\times\vec{a}| = 30$

$S \rightarrow 2$

Q.8 (ABC)

$$|\vec{x}| = |\vec{y}| = |\vec{z}| = \sqrt{2}$$

$$\theta = \frac{\pi}{3}$$

$$\vec{a} = \lambda\vec{x}\times\left(\vec{y}\times\vec{z}\right)$$

$$\vec{b} = \mu\vec{y}\times\left(\vec{z}\times\vec{x}\right)$$

$$\vec{a} = \left(\left(\vec{x}.\vec{z}\right)\vec{y} - \left(\vec{x}.\vec{y}\right)\vec{z}\right)$$

$$\vec{a} = \lambda\left(2\times\frac{1}{2}\vec{y} - 2\times\frac{1}{2}\vec{z}\right)$$

$$\vec{a} = \lambda\left(\vec{y}-\vec{z}\right)$$

$\vec{b} = \mu\left(\vec{z} - \vec{x}\right)$

Similarly

$\vec{a}.\vec{y} = \lambda\left(2 - 2 \times \frac{1}{2}\right) = \lambda$

$\vec{a} = \left(\vec{a}.\vec{y}\right)\left(\vec{y} - \vec{z}\right) \Rightarrow$ B)

$\vec{b}.\vec{z} = \mu\left(2 - 2 \times \frac{1}{2}\right)$

$\mu = \vec{b}.\vec{z}$

$\therefore\ \vec{b} = \left(\vec{b}.\vec{z}\right)\left(\vec{z} - \vec{x}\right) \Rightarrow$ (A)

(A) $\vec{a}.\vec{b} = \left(\vec{a}.\vec{y}\right)\left(\vec{y} - \vec{z}\right).\left(\vec{b}.\vec{y}\right)\left(\vec{z} - \vec{x}\right)$

$= \left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right)\left(\vec{y}\vec{z} - \vec{y}\vec{x} - 2 + \vec{x}\vec{z}\right)$

$= \left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right)$

$= -\left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right) \Rightarrow$ (C)

Q.9 [4]

$p\vec{a} + q\vec{b} + r\vec{c} = a \times b + b \times c$

Taking dot product with $\vec{a}$, $\vec{b}$, $\vec{c}$ we get

$p + \frac{q}{2} + \frac{r}{2} = [a\ b\ c]$ ......(1)

$\frac{p}{2} + q + \frac{r}{2} = 0$ ......(2)

$\frac{p}{2} + \frac{q}{2} + r = [a\ b\ c]$ ......(3)

(1) & (3) $\Rightarrow p = r$ & $q = -p$

$\frac{p^2 + 2q^2 + r^2}{q^2} = \frac{p^2 + 2p^2 + p^2}{p^2} = 4$ Ans.

Q.10 (A)

(P) $y = 4x^3 - 3x$ where $\cos\theta = x$

$\frac{dy}{dx} = 12x^2 - 3$

$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = (x^2 - 1) \,.\, 24x + x(12x^2 - 3)$

$= 36x^3 - 27x = 9(4x^3 - 3x) = 9y$

Hence $\frac{1}{y}\left\{\left(x^2 - 1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx}\right\} = 9$

(Q) $\left|\vec{a}_1 \times \vec{a}_2 + \vec{a}_2 \times \vec{a}_3 + \ldots + \vec{a}_{n-1} \times \vec{a}_n\right| =$

$\left|\vec{a}_1 \,.\, \vec{a}_2 + \vec{a}_2 \,.\, \vec{a}_3 + \ldots + \vec{a}_{n-1}.\vec{a}_n\right|$

Let $\left|\vec{a}_1\right| = \left|\vec{a}_2\right| = \ldots\ldots = \left|\vec{a}_n\right| = \lambda$

(as centre is origin)

More over angle between 2 consecutive $\vec{a}_i$'s is $\frac{2\pi}{n}$

Hence given equation reduces to

$(n - 1)\lambda^2 \sin\left(\frac{2\pi}{n}\right) = (n - 1)\lambda^2 \cos\left(\frac{2\pi}{n}\right)$

$\Rightarrow \tan\left(\frac{2\pi}{n}\right) = 1 \Rightarrow \frac{2\pi}{n} = \frac{\pi}{4} \Rightarrow n = 8$

(R) Equation of normal $\frac{6x}{h} - \frac{3y}{1} = 3$

$\left(\text{Equation of normal is } \frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2\right)$

slope $= \frac{6}{3h} = 1$ (as it is perpendicular to z + y = 1)

$\Rightarrow h = 2$

(S) $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) + \tan^{-1}\left(\frac{2}{x^2}\right)$

$$\Rightarrow \frac{\frac{1}{2x+1} + \frac{1}{4x+1}}{1 - \frac{1}{(2x+1)(4x+1)}} = \frac{2}{x^2} \Rightarrow \frac{6x+2}{8x^2+6x} = \frac{2}{x^2}$$

$\Rightarrow 3x^3 + x^2 = 8x^2 + 6x \Rightarrow\quad 3x^3 - 7x^2 - 6x = 0$

$\Rightarrow 3x^2 - 7x + 6 = 0$ (as $x \neq 0$)

$\Rightarrow (x - 3)(3x + 2) = 0 \Rightarrow x = -\frac{2}{3}, 3$

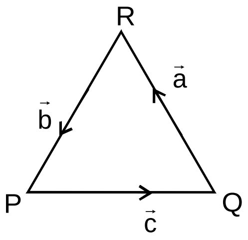
$\left(-\frac{2}{3} \text{ is rejected}\right)$

Q.11 (A,C,D)

$\vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow \vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow 48 + \vec{c}^{\,2} + 48 = 144$



$\Rightarrow \vec{c}^{\,2} = 48$

$\Rightarrow \left|\vec{c}^{\,2}\right| = 4\sqrt{3}$

$\therefore \dfrac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = 24 - 12 = 12$ **Ans. (A)**

Further

$\vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow 144 + 48 + 2\vec{a}.\vec{b} = 48$

$\Rightarrow \vec{a}.\vec{b} = -72$ **Ans. (D)**

$\because \vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow \vec{a}\times\vec{b}+\vec{a}\times\vec{c} = 0$

$\therefore \left|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}\right| = 2\left|\vec{a}\times\vec{b}\right| = 2.\sqrt{144.48-(72)^2} =$

$48\sqrt{3}$ **Ans. (C)**

Q.12 (A) → P,Q ; (B) → P, Q ; (C) → P,Q,S,T ; (D) → Q, T

(A) $\left|\left(\alpha\hat{i}+\beta\hat{j}\right).\left(\dfrac{\sqrt{3}\hat{i}+\hat{j}}{2}\right)\right| = \sqrt{3} \Rightarrow$

$\sqrt{3}\alpha+\beta = \pm 2\sqrt{3}$

$\sqrt{3}\,\alpha + \left(\dfrac{\alpha-2}{\sqrt{3}}\right) = \pm 2\sqrt{3}$

$\Rightarrow 3\alpha + \alpha - 2 = \pm 6 \Rightarrow 4\alpha = 8, -4 \Rightarrow \alpha = 2, -1$

(A → P, Q)

(B) Continuous $\Rightarrow -3a - 2 = b + a^2$

differentiable $\Rightarrow -6a = b \Rightarrow 6a = a^2 + 3a + 2$

$\Rightarrow a^2 - 3a + 2 = 0 \Rightarrow a = 1, 2$

(B → P, Q)

(D) $\dfrac{2ab}{a+b} = 4 \Rightarrow ab = 2a + 2b$ ......(i)

$q = 10 - a$ and $2q = 5 + b$

$\Rightarrow 20 - 2a = 5 + b \Rightarrow$ $15 = 2a + b$ ....(ii)

From (i) and (ii) $a(15 - 2a) = 2a + 2(15 - 2a)$

$\Rightarrow 15a - 2a^2 = -2a + 30$ $\Rightarrow 2a^2 - 17a + 30 = 0$

$\Rightarrow a = 6, \dfrac{5}{2}$

$\Rightarrow q = 4, \dfrac{15}{2}$ $\Rightarrow |q - a| = 2, 5$

(D → Q, T)

(C) Let

$a = 3 - 3\omega + 2\omega^2$

$a\omega = 3\omega - 3\omega^2 + 2$

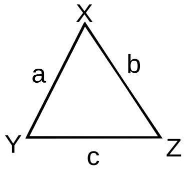
$a\omega^2 = 3\omega^2 - 3 + 2\omega$

Now $a^{4n+3}(1 + \omega^{4x+3} + (\omega^2)^{4n+3}) = 0$

$\Rightarrow$ n should not be a multiple of 3

Hence P, Q, S, T

Q.13 (A) → P,R,S ; (B) → P ; (C) → P,Q ; (D) → S, T



Given $2(a^2 - b^2) = c^2$

$\Rightarrow 2(\sin^2x - \sin^2y) = \sin^2z$

$\Rightarrow 2\sin(x + y)\sin(x - y) = \sin^2z$

$\Rightarrow 2\sin(\pi - z)\sin(x - y) = \sin^2z$

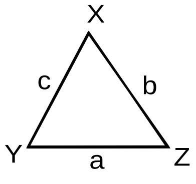
$z \Rightarrow \sin(x - y) = \dfrac{\sin z}{2}$ ...(i)

also given, $\lambda = \dfrac{\sin(x-y)}{\sin z} = \dfrac{1}{2}$

Now, $\cos(n\pi\lambda) = 0$ $\Rightarrow \cos\left(\dfrac{n\pi}{2}\right) = 0$

$\therefore$ n = 1, 3, 5 $\therefore$ (A → P,R,S)

(B)



$1 + \cos2X - 2\cos2Y = 2\sin X \sin Y$

$2\cos^2X - 2\cos2Y = 2\sin X \sin Y$

$1 - \sin^2X - 1 + 2\sin^2Y = \sin X \sin Y$

$\sin^2X + \sin X \sin Y = 2\sin^2Y$

$\sin(\sin X + \sin Y) = 2\sin^2Y$ $\sin X = ak$, $\sin Y = bk$

$a(a + b) = 2b^2$

$a^2 + ab - 2b^2 = 0$

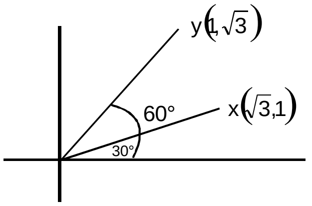
$\left(\dfrac{a}{b}\right)^2 + \dfrac{a}{b} - 2 = 0$

$\frac{a}{b}$ = –2, 1

$\frac{a}{b}$ = 1**(B → P)**

(C) Hence equation of acute angle bisector of OX and OY is y = x
Hence x – y = 0

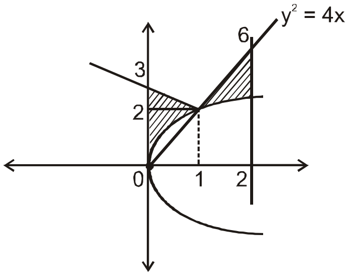
Now, distance of $\beta\hat{i}+(1-\beta)\hat{j}$ ≡ z(β, 1 – β) from x – y is $\left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$



|2β – 1| = 3
2β – 1 = ±3
2β = 4, –2
β = 2, –1
|β| = 2, 1 **Ans.** (P,Q)

(D) For α = 1

$$y = |x-1| + |x-2| + x = \begin{cases} 3-x & ; \quad x<1 \\ 1+x & ; \quad 1\le x<2 \\ 3x-3 & ; \quad x\ge 2 \end{cases}$$

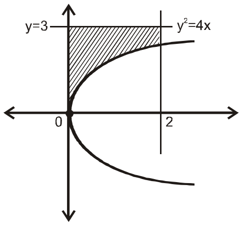


$$A = \frac{1}{2}(2+3)\times 1+\frac{1}{2}(2+3)\times 1-\int_0^2 2\sqrt{x}dx$$

$$A = 5-\frac{8}{3}\sqrt{2}$$

$\therefore$ F(1) + $\frac{8}{3}\sqrt{2}$ = 5

For α = 0, y = |–1| + |–2| = 3



$$A = 6-\int_0^2 2\sqrt{x}dx \Rightarrow A = 6-\frac{8}{3}\sqrt{2}$$

$\therefore$ F(0) + $\frac{8}{3}\sqrt{2}$ = 6 $\therefore$ (D → s, t)

Q.14 (BONUS)
This question in seem to be wrong but examiner may think like this

$$\vec{S}=4\vec{p}+3\vec{q}+5\vec{r}$$

$$\vec{S}=x(-\vec{p}+\vec{q}+\vec{r})+y(\vec{p}-\vec{q}+\vec{r})+z(-\vec{p}-\vec{q}+\vec{r})$$

–x + y – z = 4 .....(1)
x – y – z = 3 .....(2)
x + y + z = 5 .....(3)
add (1) and (2)

$-2z = 7 \Rightarrow z = -\frac{7}{2}$

2x = 8 ⇒ x = 4
y + z = 1
2x + y + z = 2(4) + 1 = 9

Q.15 (B, C)

$\hat{w}.(\hat{u}\times\vec{v})=1$

$\Rightarrow|\vec{w}||\vec{u}\times\vec{v}|\cos\theta=1$ θ is angle between $\hat{w}$ and $\hat{u}\times\vec{v}$

$\Rightarrow\cos\theta=1$ $\Rightarrow\theta=0\Rightarrow\hat{u}\times\vec{v}$ is parallel to $\hat{w}$

$\Rightarrow \hat{w}.\hat{u}=0$ *and* $\hat{w}.\vec{v}=0$

So, there are infinitely many choices for such $\vec{v}$

If $\hat{u}$ lies in the xy-plane then

$\frac{1}{\sqrt{6}}u_1+\frac{1}{\sqrt{6}}u_2=0$ $\Rightarrow u_1=-u_2$ $\Rightarrow|u_1|=|u_2|$

If $\hat{u}$ lies in the xz-plane then

$\frac{1}{\sqrt{6}}u_1+\frac{2}{\sqrt{6}}u_3=0$ $\Rightarrow u_1=-2u_3$ $\Rightarrow|u_1|=2|u_3|$

Q.16 (B,C,D)
P(3, 0, 0), R(0, 3, 0), Q(3, 3, 0), T(3/2, 3/2, 0), S(3/2, 3/2, 3)

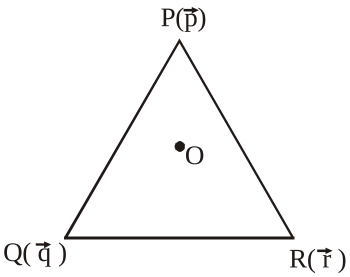
$\overrightarrow{OQ}=3i+3j$, $\overrightarrow{OS}=\frac{3}{2}i+\frac{3}{2}j+3k$

Let , θ be angle between $\overrightarrow{OQ}$ and $\overrightarrow{OS}$, then

$\cos\theta=\frac{1}{\sqrt{3}}$

Eqⁿ of plane containing triangle OQS is $x-y=0$
Hence, (b, c, d)

Q.17 (B)



$$\vec{p}.\vec{q}+\vec{r}.\vec{s}=\vec{r}.\vec{p}+\vec{q}.\vec{s}=\vec{q}.\vec{r}+\vec{p}.\vec{s}$$

$$\Rightarrow\ \vec{p}\left(\vec{q}-\vec{r}\right)-\vec{s}.\left(\vec{q}-\vec{r}\right)=0\Rightarrow\overrightarrow{PS}\ .\ \overrightarrow{QR}=0$$

Similarky $\overrightarrow{PQ}\ .\ \overrightarrow{SR}=0$

$\Rightarrow$ S is orthocentre of the triangle

Q.18 [3]

$\vec{c}=x\vec{a}+y\vec{b}+\vec{a}\times\vec{b}$

$\vec{c}.\vec{a}=x$ and $x=2\cos\alpha$

$\vec{c}.\vec{b}=x$ and $y=2\cos\alpha$

Also, $|\vec{a}\times\vec{b}|=1$

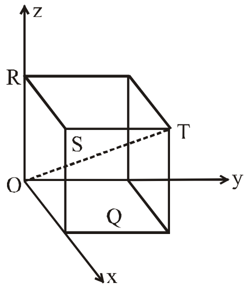
$\therefore\quad \vec{c}=2\cos(\vec{a}+\vec{b})+\vec{a}\times\vec{b}$

$\vec{c}^2=4\cos^2(\vec{a}+\vec{b})^2+2\cos\alpha(\vec{a}\times\vec{b}).(\vec{a}\times\vec{b})$

$4=8\cos^2\alpha+1$

$8\cos^2\alpha=3$

Q.19 [0.5]



$$\vec{p}=\overrightarrow{SP}=\left(\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right)=\frac{1}{2}\left(\hat{i}+\hat{j}-\hat{k}\right)$$

$$\vec{q}=\overrightarrow{SQ}=\left(-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)=\frac{1}{2}\left(-\hat{i}+\hat{j}+\hat{k}\right)$$

$$\vec{r}=\overrightarrow{SR}=\left(-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right)=\frac{1}{2}\left(-\hat{i}-\hat{j}-\hat{k}\right)$$

$$\hat{t}=\overrightarrow{ST}=\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)=\frac{1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\left|\left(\vec{p}\times\vec{q}\right)\times\left(\vec{r}\times\vec{t}\right)\right|=\frac{1}{4}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&-1\\-1&1&-1\end{vmatrix}\times\frac{1}{4}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&-1&1\\1&1&1\end{vmatrix}$$

$$=\frac{1}{16}\left|\left(2\hat{i}+2\hat{j}\right)\times\left(-2\hat{i}\times2\hat{j}\right)\right|=\left|\frac{\hat{k}}{2}\right|=\frac{1}{2}$$

Q.20 [0.75]

A(1,0,0), B $\left(\frac{1}{2},\frac{1}{2},0\right)$ & C $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Hence, $\overrightarrow{AB}=-\frac{1}{2}\hat{i}+\frac{1}{2}\hat{j}$ &

$\overrightarrow{AC}=-\frac{2}{3}\hat{i}+\frac{1}{3}\hat{j}+\frac{1}{3}\hat{k}$

So, $\Delta=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\sqrt{\frac{1}{2}\times\frac{2}{3}-\frac{1}{4}}=\frac{1}{2\times2\sqrt{3}}$

$\Rightarrow(6\Delta)^2=\frac{3}{4}=[0.75]$

Q.21 [18.00]

$$\vec{c}=(2\alpha+\beta)\hat{i}+\hat{j}(\alpha+2\beta)+\hat{k}(\beta-\alpha)$$

$$\frac{\vec{c}.(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|}=3\sqrt{2}$$

$\Rightarrow\alpha+\beta=2$ ....(1)

$(\vec{c}-(\vec{a}\times\vec{b})).(\alpha\vec{a}+\beta\vec{b})$

$=|\vec{c}|^2=\alpha^2|a|^2+\beta^2|b|^2+2\alpha\beta(\vec{a}.\vec{b})$

$=6(\alpha^2+\beta^2+\alpha\beta)$

$=6(\alpha^2+(2-\alpha)^2+\alpha(2-\alpha))$

$=6((\alpha-1)^2+3)$

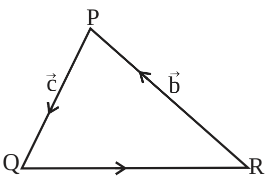
$\Rightarrow$ Min. value = 18

Q.22 [108.00]

We have, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow\vec{c}=-\vec{a}-\vec{b}$

Now, $\dfrac{\vec{a}.\left(-\vec{a}-2\vec{b}\right)}{\left(-\vec{a}-\vec{b}\right).\left(\vec{a}-\vec{b}\right)}=\frac{3}{7}$

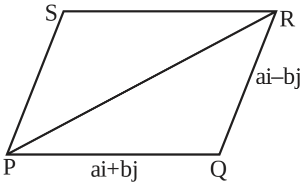


$\Rightarrow\dfrac{9+2\vec{a}.\vec{b}}{9-16}=\frac{3}{7}$

$$\Rightarrow \vec{a}.\vec{b} = -6$$

$$\Rightarrow \left|\vec{a}\times\vec{b}\right|^2 = a^2b^2 - \left(\vec{a}.\vec{b}\right)^2 = 9 \times 16 - 36 = 108$$

Q.23 (A,C)



$$\vec{u} = \left(\left(i+j\right).PQ\right)PQ$$

$$\vec{u} = \left|\left(i+j\right).PQ\right|$$

$$\left|\vec{u}\right| = \left|\left(i+j\right)\cdot\frac{\left(ai+bj\right)}{\sqrt{a^2+b^2}}\right| = \frac{a+b}{a^2+b^2}$$

$$\vec{v} = \left(i+j\right).PS$$

$$\left|\vec{v}\right| = \left|\frac{\left(i+j\right).\left(ai-bj\right)}{\sqrt{a^2+b^2}}\right| = \frac{a-b}{\sqrt{a^2+b^2}}$$

$$\left|\vec{u}\right| = \left|\vec{v}\right| = \left|\vec{w}\right|$$

$$\frac{\left|\left(a+b\right)\right| + \left|a-b\right|}{\sqrt{a^2+b^2}} = \sqrt{2}$$

For a > b

$$2a = \sqrt{2}.\sqrt{a^2+b^2}$$

$$4a^2 = 2a^2 + 2b^2$$

$a^2 = b^2 \therefore a = b$ ....(1)

(a>0, b>0)

similarly for a a>b we will get a=b

Now area of parallelogram $= \left|\left(ai+bj\right)\times\left(ai-bj\right)\right|$

= 2ab

∴ 2ab = 8

ab=4 ....(2)

from (1) and (2)

a=2, b=2 ∴ a+b=4 option (A)

length of diagonal is

$$\left|2a\hat{i}\right| = \left|4\hat{i}\right| = 4$$

so option (C)

Q.24 (A,B,C)

$$\overrightarrow{OB}\times\overrightarrow{OC} = \frac{1}{2}\overrightarrow{OB}\times\left(\overrightarrow{OB} - \lambda\overrightarrow{OA}\right)$$

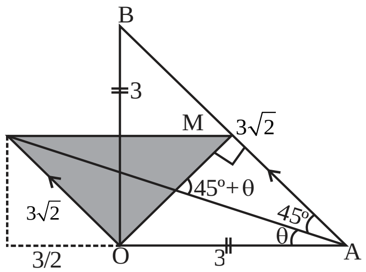
$$= \frac{\lambda}{2}\left(\overrightarrow{OA}\times\overrightarrow{OB}\right)$$

$$\left|\overrightarrow{OB}\right|\times\left|\overrightarrow{OC}\right| = \frac{\left|\lambda\right|}{2}\left|\overrightarrow{OA}\right|\times\left|\overrightarrow{OB}\right|$$

(Note $\overrightarrow{OA}$ & $\overrightarrow{OB}$ are perpendicular)

$\Rightarrow \frac{9\lambda}{2} = \frac{9}{2} \Rightarrow \lambda = 1$ (given λ>0)

So $\overrightarrow{OC} = \frac{\overrightarrow{OB} - \overrightarrow{OA}}{2} = \frac{\overrightarrow{AB}}{2}$



M is mid point of AB

Note projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ $= -\frac{3}{2}$

$$\tan\theta = \frac{1}{3}$$

Area of $\Delta ABC = \frac{9}{2}$

Acute angle between diagonals is

$$\tan^{-1}\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right) = \tan^{-1}2$$

3-Dimensional Geometry

EXERCISES

ELEMENTRY

Q.1 (2)

From x-axis $= \sqrt{y^2 + z^2} = \sqrt{4+9} = \sqrt{13}$

From y-axis $= \sqrt{1+9} = \sqrt{10}$

From z-axis $= \sqrt{1+4} = \sqrt{5}$

Q.2 (3)

Check option (3), $\dfrac{4-(-2)}{-3-4} \neq \dfrac{-3-4}{-2-(-3)}$

Therefore, this set of points is non-collinear.

Q.3 (2)

Obviously, the projection

$$= [2-(-1)]\frac{6}{7} + [5-0]\frac{2}{7} + [1-3]\frac{6}{7}$$

$$= \frac{18+10-6}{7} = \frac{22}{7}$$

Q.4 (2)

Let $A = (1,1,1)$; $B = (-2,4,1)$; $C = (-1,5,5)$ &

$D = (2,2,5)$ $AB = \sqrt{9+9+0} = 3\sqrt{2}$, $BC = \sqrt{1+1+16} = 3\sqrt{2}$

and $CD = 3\sqrt{2}$ and $AD = 3\sqrt{2}$. Hence it is a square.

Q.5 (1)

For D'ratio (1, - 3, 2), the direction cosine will be

$$\left(\frac{1}{\sqrt{1+9+4}}, \frac{-3}{\sqrt{1+9+4}}, \frac{2}{\sqrt{1+9+4}}\right)$$

$$\Rightarrow \left(\frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right)$$

Q.6 (1)

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow \cos\gamma = \sqrt{1 - \left(\frac{14}{15}\right)^2 - \left(\frac{1}{3}\right)^2}$$

$$= \sqrt{\frac{8}{9} - \left(\frac{196}{225}\right)} = \pm\frac{2}{15}$$

Q.7 (1)

If $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the d.c's

of line then, $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + n^2 = 1$

$$\Rightarrow n^2 = \frac{23}{36} \quad \Rightarrow n = \frac{\sqrt{23}}{6}$$

Q.8 (2)

Since $\alpha = \beta = \gamma \Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$$\Rightarrow \alpha = \cos^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$$

So, there are four lines whose direction cosines are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right),$$

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$$

Q.9 (1)

$$\theta = \cos^{-1}\left(\frac{3+0-5}{\sqrt{1+1}\sqrt{9+16+25}}\right)$$

$$= \cos^{-1}\left(\frac{-2}{\pm 10}\right) = \cos^{-1}\left(\frac{1}{5}\right)$$

Q.10 (3)

Projection of [(1, 2, 3) – (6, 7, 7)] along line

$$= \frac{-15-10+8}{\sqrt{17}} = \frac{-17}{\sqrt{17}}$$

Distance $= \sqrt{(5^2 + 5^2 + 4^2) - 17} = \sqrt{49} = 7$

Q.11 (3)

The perpendicular distance of (2, 4, – 1) from the line

$\dfrac{x+5}{1} = \dfrac{y+3}{4} = \dfrac{z-6}{-9}$ is

$\{(2+5)^2 + (4+3)^2 + (-1-6)$

$$-\left[\frac{1(2+5) + 4(4+3) - 9(-1-6)}{\sqrt{1+16+81}}\right]^2\Bigg\}^{1/2} = \sqrt{147 - \left(\frac{98}{\sqrt{98}}\right)^2}$$

$$= \sqrt{147-98} = \sqrt{49} = 7$$

Q.12 (4)

Since $2(1)+2(2)+(-2)(3)=0.$

Hence lines are intersecting at right angles.

Q.13 (1)

$$\theta=\cos^{-1}\frac{(-15-48+65)}{\sqrt{25+144+169}\sqrt{9+16+25}}$$

$$=\cos^{-1}\left(\frac{2}{(13\sqrt{2})(5\sqrt{2})}\right)=\cos^{-1}\left(\frac{1}{65}\right)$$

Q.14 (4)

$$\text{S.D.}=\frac{\begin{vmatrix}6 & 15 & -3\\ 3 & -1 & 1\\ -3 & 2 & 4\end{vmatrix}}{\sqrt{(-4-2)^2+(12+3)^2+(6-3)^2}}$$

$$=\frac{270}{\sqrt{270}}=\sqrt{270}=3\sqrt{30}$$

Q.15 (1)

Given lines are,

$$\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}=r_1\text{ , (say)}$$

$$\text{and }\frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}=r_2\text{ , (say)}$$

$x=3r_1+5=-36r_2-3$, $y=-r_1+7=3+2r_2$ and

$z=r_1-2=4r_2+6$

On solving, we get $x=21, y=\frac{5}{3}, z=\frac{10}{3}$.

Trick: Check through options.

Q.16 (1)

If l, m, n are direction ratios of line, then by $Al+Bm+Cn=0$

For $x-y+z-5=0,\ l-m+n=0$ …..(i)

For $x-3y-6=0,\ l-3m+0n=0$ …..(ii)

$$\text{or }\frac{l}{0+3}=\frac{m}{1-0}=\frac{n}{-3+1}\text{ or }\frac{l}{3}=\frac{m}{1}=\frac{n}{-2}$$

Direction ratios are $(3, 1, -2)$.

Note : Option (3), may also be an answer but best answer is because in (3) direction cosines are written.

Q.17 (1)

Line joining the points (3, 5, –7) and (–2, 1, 8) is,

$$\frac{x-3}{(-2)-(3)}=\frac{y-5}{(1)-(5)}=\frac{z-(-7)}{8-(-7)}$$

$$\frac{x-3}{-5}=\frac{y-5}{-4}=\frac{z+7}{15}=K\text{ , (Let)}$$

…..(i)

$\therefore$ $x=-5K+3, y=-4K+5, z=15K-7$

$\because$ Line (i) meets the yz-plane

$\therefore$ $-5K+3=0 \Rightarrow K=3/5$

Put the value of K in x, y, z

So the required point is (0, 13/5, 2).

Q.18 (1, 4)

The direction cosines of the normal to the plane are

$$\frac{1}{\sqrt{1^2+2^2+3^2}},\frac{2}{\sqrt{1^2+2^2+3^2}},\frac{-3}{\sqrt{1^2+2^2+3^2}}$$

$$\text{i.e., }\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{-3}{\sqrt{14}}$$

But $x+2y-3z+4=0$ can be written as

$-x-2y+3z-4=0$.

Thus the direction cosines are

$$-\frac{1}{\sqrt{4}},\frac{-2}{\sqrt{14}},\frac{3}{\sqrt{14}}$$

Q.19 (2)

Equation of plane parallel to y-axis is, $ax+bz+1=0$

Also $2a+1=0 \Rightarrow a=-\frac{1}{2}$ and $3b+1=0 \Rightarrow b=-\frac{1}{3}$

$\therefore$ $3x+2z=6$

Aliter : Equation of plane $\frac{x}{2}+\frac{z}{3}=1 \Rightarrow 3x+2z=6$

Q.20 (3)

Equation is $\frac{x}{-6}+\frac{y}{3}+\frac{z}{4}=1$ or $-2x+4y+3z=12$

$\therefore$ Length of perpendicular from origin

$$=\frac{12}{\sqrt{4+16+9}}=\frac{12}{\sqrt{29}}$$

Q.21 (2)

Given, equaiton of plane is passing through the point (–1, 3, 2)

$\therefore$ $A(x+1)+B(y-3)+C(z-2)=0$ .....(i)

Since plane (i) is perpendicular to each of the planes

$x+2y+3z=5$ and $3x+3y+z=0$

So, $A+2B+3C=0$ and $3A+3B+C=0$

$\therefore \frac{A}{2-9}=\frac{B}{9-1}=\frac{C}{3-6}=K$

$\Rightarrow A=-7K, B=8K, C=-3K$

Put the values of A, B and C in (i)

we get, $7x-8y+3z+25=0$, which is the required equation of the plane.

Q.22 (3)

$\sqrt{1+1+1}.\left(\frac{1-1+1+k}{\sqrt{3}}\right)=\pm 5$ and $k=\pm 5-1=4, -6$

Q.23 (3)

Equation of plane parallel to $x-2y+2z=5$ is

$x-2y+2z+k=0$ …...(i)

Now, according to question, $\frac{1-4+6+k}{\sqrt{9}}=\pm 1$

or $k+3=\pm 3 \Rightarrow k=0$ or -6

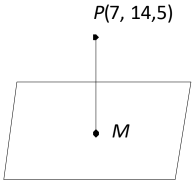
$\therefore x-2y+2z-6=0$ or $x-2y+2z=6$.

Q.24 (4)

Let M be the foot of perpendicular from (7, 14, 5) to the given plane, then PM is normal to the plane. So, its d.r.'s are 2, 4, -1. Since PM passes through $P(7,14,5)$ and has d.r.'s 2, 4, -1.

Therefore, its equation is $\frac{x-7}{2}=\frac{y-14}{4}=\frac{z-5}{-1}=r$

(Say)



$\Rightarrow x=2r+7$, $y=4r+14$, $z=-r+5$

Let co-ordinates of M be $(2r+7, 4r+14, -r+5)$

Since M lies on the plane $2x+4y-z=2$, therefore

$\Rightarrow 2(2r+7)+4(4r+14)-(-r+5)=2$

$\Rightarrow r=-3$

So, co-ordinates of foot of perpependicular are $M(1,2,8)$

Now, PM = Length of perpendicular from P

$=\sqrt{(1-7)^2+(2-14)^2+(8-5)^2}=3\sqrt{21}.$

Q.25 (1)

Obviously, $3\times 4+(-2)\times 3+2\times(-k)=0$

$\Rightarrow 12-6-2k=0 \Rightarrow k=3.$

Q.26 (2)

Equation of required plane is, $\frac{x}{1}+\frac{y}{1}+\frac{z}{1}=1$

$\Rightarrow x+y+z=1$

Q.27 (4)

The plane will be $x+2y+4z=2\times 1+3\times 2+4\times 4$

or $x+2y+4z=24$.

Q.28 (1)

If plane $x-3y+5z=d$ passes through point (1, 2, 4).

Then $1-6+20=d \Rightarrow d=15$

$\therefore$ Plane, $x-3y+5z=15$, $\Rightarrow \frac{x}{15}+\frac{y}{-5}+\frac{z}{3}=1$

Hence length of intercept cut by it on the axes (x, y, z) are respectively (15, - 5, 3).

Q.29 (3)

$\frac{5x}{60}-\frac{3y}{60}+\frac{6z}{60}=1 \Rightarrow \frac{x}{12}-\frac{y}{20}+\frac{z}{10}=1$

Hence, the intercepts are (12, – 20, 10).

Q.30 (1)

Equation of plane passing through the point (1, 2, 3) is

$A(x-1)+B(y-2)+C(z-3)=0$ .....(i)

Plane (i) is parallel to plane $x+2y+5z=0$

$\therefore (x-1)+2(y-2)+5(z-3)=0$ is the required plane.

Q.31 (2)

Equation of plane passing through the intersection of given planes, is

$(x+2y+3z+4)+\lambda(4x+3y+2z+1)=0$ .....(i)

Plane (i) is passing through origin i.e., (0, 0, 0)

$\therefore 4+\lambda=0 \Rightarrow \lambda=-4$

Put the value of λ in (i),

$-15x-10y-5z=0 \Rightarrow 3x+2y+z=0.$

Q.32 (1)

Distance of plane from origin

$=\frac{d}{\sqrt{a^2+b^2+c^2}}=\frac{14}{\sqrt{6^2+3^2+2^2}}=\frac{14}{\sqrt{49}}=\frac{14}{7}=2$

Q.33 (2)

Any plane passing through (1, 1, 1) is

$a(x-1)+b(y-1)+c(z-1)=0$ .....(i)

Plane (i) is also passing through (1, –1, –1)

$\therefore a.0+b(-2)+c(-2)=0$

or, $0.a-2b-2c=0$

or $0.a-b-c=0$ ......(ii)

Plane (i) is perpendicular to $2x-y+z+5=0$

So, $2a-b+c=0$ .....(iii)

From (ii) and (iii), $a=b=1, c=-1$

Substituting in (i) we have $x+y-z-1=0$.

Q.34 (1)

The equation of the plane through the intersection of the plane x + y + z = 1 and 2x + 3y – z + 4 = 0 is

$(x+y+z-1)+\lambda(2x+3y-z+4)=0$

or $(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z+4\lambda-1=0$

Since the plane parallel to x-axis,

$\therefore\ 1+2\lambda=0 \Rightarrow \lambda=-\frac{1}{2}$

Hence, the required equation will be $y-3z+6=0$.

Q.35 (4)

Equation of any plane passing through (0, 1, 2) is

$a(x-0)+b(y-1)+c(z-2)=0$ .....(i)

Plane (i) passes through (–1, 0, 3), then

$a(-1-0)+b(0-1)+c(3-2)=0$

$\Rightarrow -a-b+c=0 \Rightarrow a+b-c=0$ .....(ii)

Plane (i) is perpendicular to the plane

$2x+3y+z=5$, then $2a+3b+c=0$ .....(iii)

Solving (ii) and (iii), we get a = – 4k, b = 3k, c = – k

Putting these values in (i),

$-4k(x)+3k(y-1)-k(z-2)=0$

$\Rightarrow -4x+3y-3-z+2=0$

$\Rightarrow -4x+3y-z-1=0$

$\Rightarrow 4x-3y+z+1=0$

Q.36 (4)

$\frac{x}{1}=\frac{y-1}{2}=\frac{z+2}{3}=r$, (say)

So, $x=r, y=2r+1, z=3r-2$

$\therefore\ 2r+3(2r+1)+(3r-2)=0 \Rightarrow r=\frac{-1}{11}$

Hence, $x=\frac{-1}{11}, y=\frac{9}{11}, z=-\frac{25}{11}$

Q.37 (4)

Equation of plane passing through the point (1, 0, –1) is,

$a(x-1)+b(y-0)+c(z+1)=0$ ......(i)

Also, plane (i) is passing through (3, 2, 2)

$\therefore\ a(3-1)+b(2-0)+c(2+1)=0$

or $2a+2b+3c=0$ …..(i)

Plane (i) is also parallel to the line $\frac{x-1}{2}=\frac{y-1}{-2}=\frac{z-2}{3}$

$\therefore\ 2a-2b+3c=0$ …..(ii)

From (i) and (ii), $\frac{a}{-3}=\frac{b}{0}=\frac{c}{2}$

Therefore, the required plane is,

$-3(x-1)+0(y-0)+2(z+1)=0$

or $-3x+2z+5=0$.

Q.38 (2)

Line is $\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}=\lambda$ (Let)

$x=3\lambda-3;\ y=-2\lambda+2;\ z=\lambda-1$ line intersects plane,

therefore, $4(3\lambda-3)+5(-2\lambda+2)+3(\lambda-1)-5=0$

$\Rightarrow \lambda=2$. So, $x=3;\ y=-2;\ z=1$.

Trick : Since the point (3, –2, 1) satisfies both the equations.

Q.39 (3)

The equation of plane containing the line

$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ is

$a(x+1)+b(y-3)+c(z+2)=0$ .....(i)

where $-3a+2b+c=0$ .....(ii)

This passes through (0, 7, -7)

$\therefore\ a+4b-5c=0$ ….(iii)

From (ii) and (iii), $\frac{a}{-14}=\frac{b}{-14}=\frac{c}{-14}$ or $\frac{a}{1}=\frac{b}{1}=\frac{c}{1}$

Thus, the required plane is $x+y+z=0$.

Q.40 (2)

Angle between the plane and line is

$$\sin\theta=\frac{aa'+bb'+cc'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}$$

Here, $aa'+bb'+cc'=2\times3+3\times2-4\times3=0$

$\therefore \sin\theta=0 \Rightarrow \theta=0^o$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

$x^2 + y^2 + y^2 + z^2 + z^2 + x^2 = 36$

$2(x^2 + y^2 + z^2) = 36$

$$\sqrt{x^2 + y^2 + z^2} = 3\sqrt{2}$$

Q.2 (3)

$PA^2 - PB^2 = 2k^2$

$(x-3)^2 + (y-4)^2 + (z-5)^2 - (x+1)^2 - (y-3)^2 - (z+7)^2 = 2k^2$

$\Rightarrow 8x + 2y + 24z + 9 + 2k^2 = 0$

Q.3 (2)

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\alpha + \beta = 90^o$

$\sin^2\beta + \cos^2\beta + \cos^2\gamma = 1$

$\alpha = 90^o - \beta$

$\cos^2\gamma = 0$ $\qquad\qquad$ $\cos\alpha = \sin\beta$

$\therefore\ \gamma = 90^o$

Q.4 (4)

$$\ell = \cos\alpha = \frac{1}{\sqrt{2}}$$

$$\mu = \cos\beta = \frac{1}{\sqrt{2}}$$

$\ell^2 + m^2 + n^2 = 1$

$n = 0 \Rightarrow \cos\gamma = 0$

$$\Rightarrow \gamma = \frac{\pi}{2}$$

Q.5 (3)

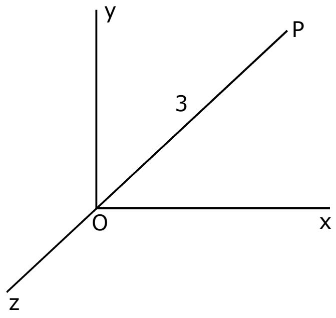
$\cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$2\cos^2\theta = 1 - \cos^2\beta = \sin^2\beta$

$2\cos^2\theta = 3\sin^2\theta = 3 - 3\cos^2\theta$

$\cos^2\theta = 3/5$

Q.6 (1)



D.R. of OP = (1, –2, –2)

$$\text{D.C. of OP} = \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$$

$$\text{Vector } \overrightarrow{OP} = |\overrightarrow{OP}|\left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right) = (1, -2, -2)$$

Q.7 (1)

Dr’s of AB = $1, -3-\alpha, 0$

Dr’s of CD = $3-\beta, 2, -2$

$AB \perp CD$

$\therefore 1(3-\beta) + (-3-\alpha) . 2 + 0 = 0$

$3 - \beta - 6 - 2\alpha = 0$

$2\alpha + \beta + 3 = 0$

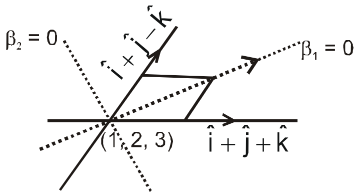
$\therefore \alpha = -1, \beta = -1$

Q.8 (1)

Dr’s of bisector

$$\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}} + \frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}} = \lambda(\hat{i}+\hat{j})$$

Hence Dr’s are $\lambda, \lambda, 0$ ($\lambda \in R$)



Equation of bisector

$$\frac{x-1}{\lambda} = \frac{y-2}{\lambda} = \frac{z-3}{0}$$

$$\frac{x-1}{2} = \frac{y-2}{2};\ z-3=0$$

Q.9 (4)

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore$ lines are perpendicular

Q.10 (1)

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -5\,\hat{i} + 5\,\hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ a & 1 & -1 \end{vmatrix} = -2\,\hat{i} + (2+3a)j + (2+a)\,\hat{k}$$

p (0, –5, –3) ; R(0, –1/5, –3/5)

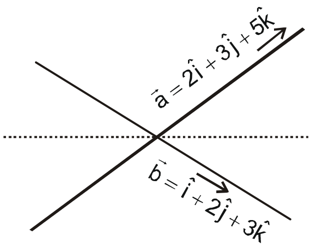
For compaire lines

$[\overrightarrow{PQ} \quad \vec{n}_1 \quad \vec{n}_2] = 0 \Rightarrow a = -2$

Q.11 (3)

$$\frac{x}{2}=\frac{y}{3}=\frac{z}{5} \qquad \text{... (i)}$$

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3} \qquad \text{... (ii)}$$



$$\hat{a}+\hat{b} = \frac{2\hat{i}+3\hat{j}+5\hat{k}}{\sqrt{38}} + \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$$

⇒ (1) and (2) will be incorrect

Let the dr's of line ⊥ to (1) and (2) be a, b, c

⇒ $2a + 3b + 5c = 0$ ... (iii)

and $a + 2b + 3c = 0$ ... (iv)

$$\therefore \frac{a}{9-10}=\frac{b}{5-6}=\frac{c}{4-3}$$

$$\Rightarrow \frac{a}{-1}=\frac{b}{-1}=\frac{c}{1}$$

$$\Rightarrow \frac{a}{1}=\frac{b}{1}=\frac{c}{-1}$$

∴ equation of line passing through (0, 0, 0) and is ⊥r to the lines (1) and (2) is

$$\frac{x}{1}=\frac{y}{1}=\frac{z}{-1}$$

Q.12 (4)

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3}=\lambda \Rightarrow \text{point } (\lambda, 2\lambda, 3\lambda)$$

$$\frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}=M$$

⇒ Point $(3M + 1, -M + 2, 4M + 3)$

$$\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}=t$$

⇒ Point $(3t - k, 2t + 1, ht + 2)$

If all three lines are concurrent

$\lambda = 3\mu + 1;\ 2\lambda = -\mu + 2;\ 3\lambda = 4\mu + 3$

$\lambda = 1 \Rightarrow \mu = 1$

$3t - k = 1;\quad 2t + 1 = 2 \Rightarrow k = \frac{1}{2} \Rightarrow t = \frac{1}{2}$

$ht + 2 = 3$

$ht = 1 \Rightarrow h = 2$

Q.13 (1)

A (a, b, c) B(a', b', c')

Line $\overrightarrow{AB}$ = (a, b, c) + λ (a' – a, b' – b, c' – c)

= (a + λa', b + λb', c + λc') – λ(a, b, c)

It will passes through origin when

a + λa' = b + λb' = c + λc' = 0

$$\Rightarrow \frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}$$

Q.14 (2)

x = y + a = z ....(1)

x + a = 2y = 2z ....(2)

we have option (2) & (3)

but if we look at option B

it will satisfy the given equation

Q.15 (3)

$$\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k};$$

$$\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$$

A(2, 3, 4),	B (1, 4, 5),
D.R. (1, 1, –k),	D.R. (k, 2, 1)

$$\text{Coplanar then} = \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

⇒ k = 0 or k = –3

Q.16 (2)

$\vec{a}$ = (2, 5, –3)

$\vec{b}$ = (–1, 8, 4)

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{-2+40-12}{\sqrt{38}\ \sqrt{81}} = \frac{26}{9\sqrt{38}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

Q.17 (2)

$$\text{dir}^n \text{ of line} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{vmatrix} = -2\hat{i} + \hat{k}$$

DR' & = (–2, 0, 1)

$(\vec{n}_1 \times \vec{n}_2) \times \hat{k} = (-2\hat{i} + \hat{k}) \times \hat{k} = 2\hat{j}$

⇒ distance = 2

Q.18 (1)

$a(x-2) + b(y+3) + c(z-1) = 0$

Dr's of the line joining (3, 4, – 1) & (2, – 1, 5) are – 1, – 5, 6

normal of the plane and above line are parallel

∴ equation of the plane

$-1(x-2) - 5(y+3) + 6(z-1) = 0$

$\Rightarrow x + 5y - 6z + 19 = 0$

Q.19 (4)

$(xy + yz) = 0$

$x + z = 0$ and $y = 0$

Two perpendicular plane.

Q.20 (1)

$x + 2y + 2z = 5$ $\vec{n}_1 = (1, 2, 2)$

$3x + 3y + 2z = 8$ $\vec{n}_2 = (3, 3, 2)$

$$\text{Normal vector of plane} = \vec{n}_1 \times \vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 3 & 2 \end{vmatrix}$$

$= -2\hat{i} + 4\hat{j} + 3\hat{k}$

Equation of plane $-2x + 4y - 3z = k$

passing through $(1, -3, -2)$ $k = -8$

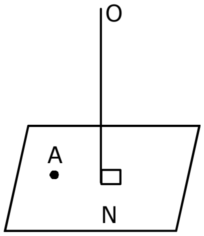
$-2x + 4y - 3z = -8$

$2x - 4y + 8z - 8 = 0$

Q.21 (1)

Let N be foot of poerpendicular = (α, β, γ)

N(α, β, γ)



A (1, 2, 3)

Equation of plane willk be

$\alpha x + \beta y + \gamma z = k$

passing through (1, 2, 3)

$\Rightarrow k = \alpha + 2\beta + 3\gamma$

$\alpha x + \beta y + \gamma z = \alpha + 2\beta + 3\gamma$

this plane passes through (α, β, γ) also

$\alpha^2 + \beta^2 + \gamma^2 = \alpha + 2\beta + 3\gamma$

$x^2 + y^2 + z^3 - x - 2y - 3z = 0$

Q.22 (2)

N (α, β, γ)

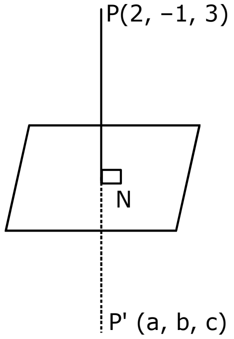
$3x - 2y - z = 9$

$$\frac{\alpha-2}{3} = \frac{\beta+1}{-2} = \frac{\gamma-3}{-1} = \lambda$$

$\alpha = 3\lambda + 2, \beta = -2\lambda - 1, \gamma = -\lambda + 3$

N point lies on the plane

$3(3\lambda + 2) - 2(-2\lambda + 1) - (-\lambda + 3) = 9$



$$\Rightarrow \lambda = \frac{2}{7}$$

$$N\left(\frac{20}{7}, \frac{-11}{7}, \frac{19}{7}\right)$$

$$N = \frac{P + P'}{2} \Rightarrow P^1 = 2N - P$$

$$\Rightarrow P^1\left(\frac{26}{7}, \frac{-15}{7}, \frac{17}{7}\right)$$

Q.23 (4)

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$$

Use pases through P(2, –1, 2)

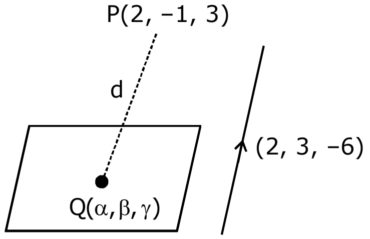
point P

So P_0I of line and plane is P (2, –1, 2)

(–1, –5, –10) so PQ = 13

Q.24 (1)

$$\frac{\alpha-1}{2} = \frac{\beta+2}{3} = \frac{\gamma-3}{-6} = \lambda$$



$\alpha = 2\lambda + 1,\ b = 3l - 2,\ \gamma = -6\lambda + 3$

(α, β, γ) lie on the plane $x + y + z = 5$

$$\Rightarrow \lambda = \frac{1}{7}$$

$$Q\left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$$

$d = PQ = 1$

Q.25 (1)

Let the Eq^n of plane

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$$

passes through (a, b, c)

$$\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 1$$

common point will be (α, β, γ)

so locus

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$$

Q.26 (1)

Let the equation of planes

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \ \& \ \frac{x}{a_1} + \frac{y}{b_1} + \frac{z}{c_1} = 1$$

perpendicular distance from orign will be same

$p_1 = p_2$

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}\right| = \left|\frac{-1}{\sqrt{\frac{1}{a_1^2} + \frac{1}{b_1^2} + \frac{1}{c_1^2}}}\right|$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a_1^2} + \frac{1}{b_1^2} + \frac{1}{c_1^2}$$

Q.27 (4)

Direction of line = (1, 2, 2)

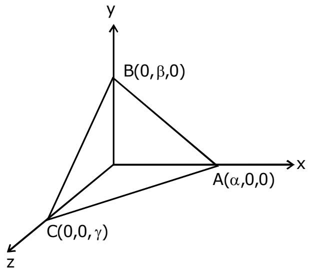
normal vector of plane = $(2, -1, \sqrt{\lambda})$

$$\sin\theta = \frac{2 - 2 + 2\sqrt{\lambda}}{\sqrt{1 + 4 + 4}\sqrt{4 + 1 + \lambda}} = \frac{1}{3}$$

$4\lambda = 5 + \lambda$

$$\lambda = \frac{5}{3}$$

Q.28 (3)



Let the equation of plane :

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1 \qquad ....(1)$$

$$\frac{\alpha}{3} = a \Rightarrow \alpha = 3a$$

$$\frac{\beta}{3} = b \Rightarrow \alpha = 3a$$

$$\frac{\gamma}{3} = c \Rightarrow \gamma = 3c$$

$$\frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$$

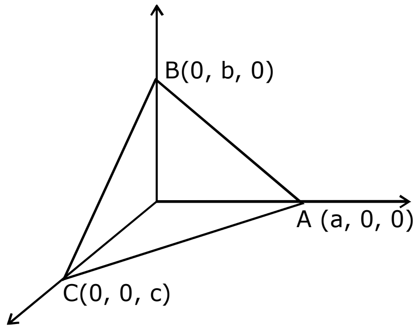
Q.29 (1)

Ley the equation of plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

given that $p = \dfrac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$

$$\text{or } \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2} \qquad ....(1)$$



Let centroid (u, v, w)

$v = \frac{b}{4} \Rightarrow b = 4v$

$w = \frac{c}{4} \Rightarrow c = 4w$

$$\frac{1}{16u^2} + \frac{1}{16v^2} + \frac{1}{16w^2} = \frac{1}{p^2}$$

$$\frac{1}{u^2} + \frac{1}{v^2} + \frac{1}{w^2} = \frac{16}{p^2}$$

$u^{-2} + v^{-2} + z^{-2} = 16p^{-2}$

Q.30 (2)

Let Point P (α, β, γ)

Given that

$(\alpha - 1^2) + (\alpha + 1)^2 + (\beta - 1) + (\beta + 1)^2 + (\gamma - 1)^2 + (\gamma + 1)^2$ $= 10$

$2\alpha^2 + 2\beta^2 + 2\gamma^2 + 6 = 0$

$\alpha^2 + \beta^2 + \gamma^2 = 2 \Rightarrow x^2 + y^2 + z^2 = 2$

Q.31 (1)

A $(2 - x, 2, 2)$ B $(2, 2 - y, 2)$ C $(2, 2, 2 - z)$ D$(1, 1, 1)$

$\overrightarrow{AB} = (x, -y, 0)$, $\overrightarrow{AC} = (x, 0, -2)$,

$\overrightarrow{AD} = (x - 1, -1, -1)$

If A, B, C, D are coplanar points then

$[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

$$\begin{vmatrix} x & -y & 0 \\ x & 0 & -2 \\ x-1 & -1 & -1 \end{vmatrix} = 0 \qquad \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

Q.32 (2)

$a(x - 1) + b(y - 0) + c(z - 0) = 0 \qquad (1, 0, 0)$

$\Rightarrow \quad ax + by + cz - a = 0 \qquad (0, 1, 0)$

$0 + b + 0 - a = 0$

$\boxed{b = a}$

$\angle$ between planes = $\angle$ between normal

$x + y = 3, \qquad (1, 1, 0) \qquad (a, b, c)$

$$\cos\frac{\pi}{4} = \frac{1(a) + 1(b) + 0(c)}{\sqrt{1^2 + 1^2 + 0^2}\ \sqrt{a^2 + b^2 + c^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{a + b}{\sqrt{2}\sqrt{a^2 + b^2 + c^2}} \qquad \boxed{b = a}$$

$\sqrt{2a^2 + c^2} = 2a$

$2a^2 + c^2 = 4a^2$

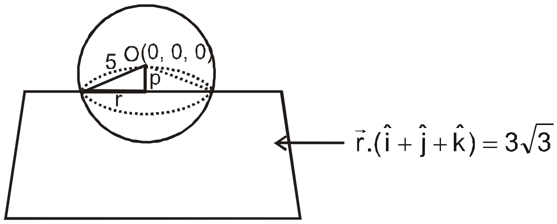
$2a^2 = c^2$

$\boxed{c = \sqrt{2}\,a}$

$b = a\ ;\ c = \sqrt{2}\,a$

$a : b : c = \left(1{:}1{:}\sqrt{2}\right)$

Q.33 (2)



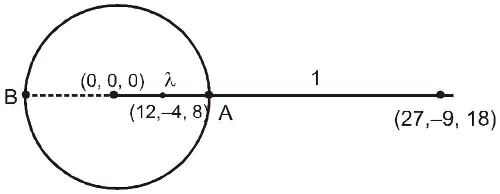
$x + y + z - 3\sqrt{3} = 0$

$$p = \left|\frac{-3\sqrt{3}}{\sqrt{3}}\right| = 3$$

$\Rightarrow r = 4$

Q.34 (1)

$$A\left(\frac{27\lambda + 12}{\lambda + 1}, \frac{-9\lambda - 4}{\lambda + 1}, \frac{18\lambda + 8}{\lambda + 1}\right)$$



Which lies on the sphere

$$\therefore \left(\frac{27\lambda + 12}{\lambda + 1}\right)^2 + \left(\frac{-9\lambda - 4}{\lambda + 1}\right)^2 + \left(\frac{18\lambda + 8}{\lambda + 1}\right)^2 = 504$$

Solving above we get $9\lambda^2 = 4$

$$\lambda = \pm\frac{2}{3}$$

Q.35 (3)

$$\frac{x - 2}{3} = \frac{y + 1}{2} = \frac{z - 1}{-1} = \lambda$$

$(3\lambda + 2, 2\lambda - 1, 1 - \lambda)$

$z = 0 \Rightarrow \lambda = 1$

$xy = c^2$

$(3\lambda + 2)(2\lambda - 1) = c^2$

put $\lambda = 1 \Rightarrow c^2 = 5 \Rightarrow c = \pm\sqrt{5}$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (C)

Distance = $\sqrt{x^2 + y^2 + z^2}$

$= \sqrt{(2t)^2 + (4t)^2 + (4t)^2}$

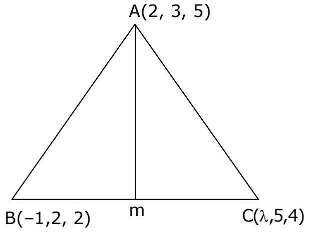
$= 6t$

$t = 10$

Distance = 60 km

Q.2 (C)

A (2, 3, 5) B(–1, 2, 2) C(λ, 5, 4)



m $\left(\frac{\lambda-1}{2}, \frac{7}{2}, \frac{\mu+2}{2}\right)$

D.R> of median through A :

$\left(\frac{\lambda-1}{2} - 2\, \frac{7}{2} - 3, \frac{\mu+2}{2} - 5\right)$

$\left(\frac{\lambda-5}{2}, \frac{1}{2}, \frac{\mu-8}{2}\right)$

As thje median through A is equally inclined to He axis

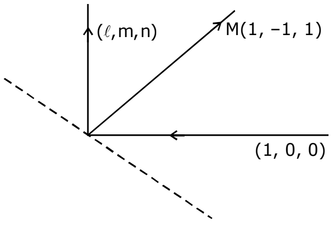
∴ D.R.'s will be and equal to k.

$\frac{\frac{\lambda-5}{2}}{k} = \frac{1}{2k} = \frac{\frac{\mu-8}{2}}{k} \qquad \Rightarrow \lambda = 6 \text{ and } \mu = 9$

Q.3 (D)

The DC's of incident RAy arew (1, 0, 0). Let the D.C's of reflectd ray be (λ, m, n)

⇒ The D.R.'s of the normal to polane of mirror is (l – 1, m, n)



$\frac{\ell-1}{1} = \frac{m}{-1} = \frac{n}{1}$

$\ell = \lambda + 1, m = -\lambda, n = \lambda$

$\ell^2 + m^2 + n^2 = 1$

$(\lambda + 1)^2 + \lambda^2 + \lambda^2 = 1$

$3\lambda^2 + 2\lambda = 0$

$\lambda = -2/3$

D.C's of reflected Ray $\left(\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$

or $\left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

Q.4 (A)

Let P be the centroid

$\therefore PA_1 = \frac{1}{3} AA_1, PB_1 = \frac{1}{3} BB_1$ and $PC_1 = \frac{1}{3} CC_1$

$\therefore \frac{PA_1}{AA_1} + \frac{PB_1}{BB_1} + \frac{PC_1}{CC_1} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

Q.5 (B)

Since three lines are mutually perpendicular

$\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0 ; \ell_2\ell_3 + m_2m_3 + n_2n_3 = 0$

$\ell_3\ell_1 + m_3m_1 + n_3n_1 = 0$

Also $\ell_1^2 + m_1^2 + n_1^2 = 1; \ell_2^2 + m_2^2 + n_2^2 = 1;$

$(\ell_1 + \ell_2 + \ell_3)^2 + (m_1 + n_2 + m_3)^2$

$+ (n_1 + n_2 + n_3)^2$

$= (\Sigma\ell_1^2 + \Sigma\ell_2^2 + \Sigma\ell_1\ell_3^2 + 2\Sigma\ell_1\ell_2$

$+ 2\Sigma\ell_2\ell_3 + 2\Sigma\ell_3\ell_1) = 3$

$\Rightarrow (\ell_1 + \ell_2 + \ell_3)^2 + (m_1 + m_2 + m_3)^2$

$+ (n_1 + n_2 + m_3)^2 = 3$

Hence direction cosines of OP are

$\left(\frac{\ell_1 + \ell_2 + \ell_3}{\sqrt{3}}, \frac{m_1 + m_2 + m_3}{\sqrt{3}}, \frac{n_1 + n_2 + n_3}{\sqrt{3}}\right)$

Q.6 (A)

Direction ratio's of line = (–2, 1, 2)

Direction cosine's = $\left(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

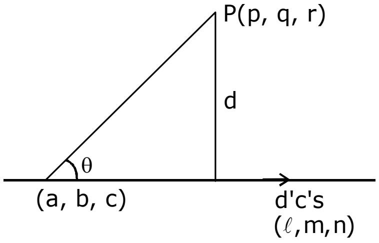
$\cos\theta = \frac{-2}{3}, \cos\theta_2 = \frac{1}{3}; \cos\theta_3 = \frac{2}{3}$

$\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$

$= 2[\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3] - 3$

$= 2\left[\frac{4}{3} + \frac{1}{3} + \frac{4}{3}\right] - 3 = -1$

Q.7 (A)



$\frac{x-a}{\ell} = \frac{y-b}{m} = \frac{z-c}{n}$ Point (p, q, r)

Let $\vec{r_1} = (p-a)\,\hat{i} + (q-b)\,\hat{j} + (r-c)\,\hat{j}$

$\vec{r_2} = \ell\,\hat{i} + m\,\hat{j} + n\,\hat{k}$

$$\cos\theta = \frac{\vec{r_1}\,.\,\vec{r_2}}{|\,\vec{r_1}\,|\,\vec{r_2}\,|}$$

also $d = |\,\vec{r_1}\,|\sin\theta$

$d^2 = |\,\vec{r_1}\,|^2\sin^2\theta$

$$= |\,\vec{r_1}\,|^2(1-\cos^2\theta) = |\,\vec{r_1}\,|^2\left[\frac{(\vec{r_1}\,.\,\vec{r_2})^2}{|\,\vec{r_1}\,|^2|\,\vec{r_2}\,|^2}\right]$$

$d^2 = |\,\vec{r_1}\,|^2 - (\vec{r_1}\cdot\vec{r_2})^2$

$= [(P-a)^2 + (q-b)^2 + (r-c)^2]$

$\quad - [\ell(p-a) + m(q-b) + n(r-c)]^2$

Q.8 (B)

$\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2} = r$ ... (1)

$\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ .... (2)

$\therefore$ coordinates of any point P on line (1)

$\therefore$ P(3r + 1, r + 2, 2r + 3)

for point of intersection of (1) and (2)

$$\frac{3r+1-3}{1} = \frac{r+2-1}{2} = \frac{2r+3-2}{3}$$

$$\frac{3r-2}{1} = \frac{r+1}{2} = \frac{2r+1}{3}$$

$\therefore r = 1$

$\therefore$ point of intersection is (4, 3, 5)

$\therefore$ the equation of required plane

$4(x-4) + 3(y-3) + 5(z-5) = 0$

$4x + 3y + 52 = 50$

Q.9 (A)

$2x - y + 3z + 4 = 0 = ax + y - z + 2$ ... (1)

$\because$ equation of plane through (1) is

$(2x - y + 3z + 4) + \lambda(ax + y - z + 2) = 0$

$x(2 + a\lambda) + y(\lambda - 1) + z(3 - \lambda) + (4 + 2\lambda) = 0$.... (2)

$x - 3y + z = 0 = x + 2y + 2 + 1$ ... (3)

$\because$ equation of plane passing through (3) is

$(x - 3y + z) + \mu(x + 2y + z + 1) = 0$

$x(1 + \mu) + y(2\mu - 3) + z(\mu + 1) + \mu = 0$.... (4)

if lines (1) and (3) are coplanar, then

$$\frac{2+a\lambda}{\mu+1} = \frac{\lambda-1}{2\mu-3} = \frac{3-\lambda}{\mu+1} = \frac{4+2\lambda}{\mu}$$

Solving this we get $\lambda = -1,\ \mu = 1$

$\therefore$ $a = -2$

Q.10 (D)

$x = ay + b,\ z = cy + d$

and $x = a'y + b',\ z = c'h + d'$

$\frac{x-b}{a} = y = \frac{z-d}{c}$

and $\frac{x-b'}{a'} = y = \frac{z-d'}{c'}$

poer pendicular then

$aa' + 1 + cc' = 0$

Q.11 (A)

Angle between two faces is equal to the angle between the normals $\vec{n_1}$ and $\vec{n_2}$.

$\vec{n_1} \rightarrow$ normal of OAB

$\vec{n_2}$ = normal of ABC

$$\vec{n_1} = \vec{OA} \times \vec{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

$= 5\,\hat{i} - \hat{j} - 3\hat{k}$ ...(1)

$$\vec{n_2} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

$= \hat{i} - 5\,\hat{j} - 3\hat{k}$ ...(2)

$$\cos\theta = \frac{\vec{n_1}\,.\,\vec{n_2}}{|\,\vec{n_2}\,||\,\vec{n_2}\,|} = \frac{19}{35} \Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

Q.12 (D)

$2x - y + z = 6$ ... (1)

$x + y + 2z = 7$ ... (2)

$x - y = 3$ ... (3)

Let the equation of plane ⊥r to (2) and (3) be

$ax + by + cz + d = 0$ .... (4)

$\therefore a + b + 2c = 0$

$a - b + 0 \,.\, c = 0$

$$\therefore \frac{a}{2} = \frac{b}{2} = \frac{c}{-1-1}$$

dr's of normal to the plane ⊥r to (2) and (3) are 2, 2, –2

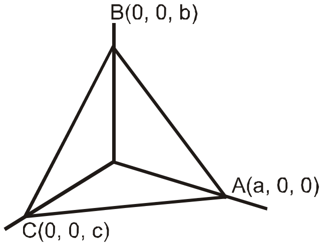
now angle between both planes is $\cos\theta$ =

$$\frac{(2)(2)+2(-1)+(-2)(1)}{3.2\sqrt{3}} = 0$$

$\Rightarrow \theta = 90°$

Q.13 (A)

Let the equation of plane be



$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

as (α, β, γ) is centroid

$$\because \alpha = \frac{a}{3} \,;\, \beta = \frac{b}{3} \text{ and } \gamma = \frac{c}{3}$$

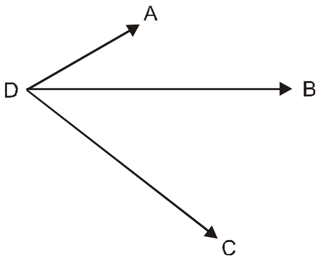
$$\therefore \text{ equation of plane } \frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$$

Q.14 (A)

A (2 – x , 2 , 2) , B (2 , 2 – y , 2) , C (2 , 2 , 2 – z), D(1 , 1 , 1)

$$\overrightarrow{DA} = (1-x)\hat{i} + \hat{j} + \hat{k}$$

$$\overrightarrow{DB} = \hat{i} + (1-y)\hat{j} + \hat{k}$$



$$\overrightarrow{DC} = \hat{i} + \hat{j} + (1-z)\,\hat{k}$$

If four points are coplanar then $[\overrightarrow{DA}, \overrightarrow{DB}, \overrightarrow{DC}] = 0$

$$\Rightarrow \begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-y & 1 \\ 1 & 1 & 1-z \end{vmatrix} = 0$$

$c_1 \to c_1 - c_2$ and $c_2 \to c_2 - c_3$

$$\Rightarrow \begin{vmatrix} -x & 0 & 1 \\ y & -y & 1 \\ 0 & z & 1-z \end{vmatrix} = 0$$

$\therefore \; -x(-y + yz - z) + 1(+yz) = 0$

$xy - xyz + xz + yz = 0$

$xy + yz + zx = xyz$

$$\therefore \; \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

Q.15 (C)

Equation of lines :

$$\frac{x-2}{3-2} = \frac{y+3}{-4+3} = \frac{z-1}{-5-1}$$

$$\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = \frac{z-1}{-6} = \lambda$$

Points $(\lambda + 2, -\lambda - 3, -6\lambda + 1)$

Point will be on given plane

$2(\lambda + 2) + (-\lambda - 3) + (-6\lambda + 1) = 7$

$\Rightarrow \lambda = -1$

Intersection point (1, –2, 7)

Q.16 (B)

Let the point be $p(\alpha, \beta, \gamma)$

$\because (\alpha - 1)^2 + (\alpha + 1)^2 + (\beta - 1)^2 + (\beta + 1)^2 + (\gamma - 1)^2 + (\gamma + 1)^2 = 10$

$\Rightarrow 2(\alpha^2 + \beta^2 + \gamma^2) = 4$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = 2$

$\therefore$ required locus is $x^2 + y^2 + z^2 = 2$

Q.17 (D)

$|\overrightarrow{AC}| = 2 \,;\, |\vec{a}| = |\vec{b}| = |\vec{a} - \vec{b}| = 4\sqrt{2}$

$|\vec{a} - \vec{b}| = 2$

$$\cos\theta = \frac{\left(\dfrac{\vec{b}}{2} - \vec{a}\right) \cdot \left(\dfrac{\vec{b} + \vec{c}}{2}\right)}{\left|\dfrac{\vec{b}}{2} - \vec{a}\right| \left|\dfrac{\vec{b} + \vec{c}}{2}\right|} = \frac{(\vec{b} - 2\vec{a}).(\vec{b} + \vec{c})}{\left|\vec{b} - 2\vec{a}\right| \left|\vec{b} - \vec{c}\right|}$$

put all the values $\cos\theta = \frac{1}{\sqrt{2}}$

Q.18 (A)

required plane $(x - y + 2z - 3) + \lambda(4x + 3y - z - 1) = 0$

$x(4\lambda + 1) + y(3\lambda - 1) + z(2 - \lambda) - (3 + \lambda) = 0$ ... (1)

Now we can observe that from the given options equation (1) can represent only (A)

Q.19 (C)

Equation of any sphere passes through the circle $x^2 + y^2 = 4$, $z = 0$ is

$x^2 + y^2 - 4 + \lambda z = 0$

its centre is $\left(0, 0, -\frac{\lambda}{2}\right)$ and radius = $\sqrt{\frac{\lambda^2}{4} + 4}$

distance of $\left(0, 0, -\frac{\lambda}{2}\right)$ from the plane x + 2y + 2z = 0 is

$$\left|\frac{2\left(-\frac{\lambda}{2}\right)}{\sqrt{1+4+4}}\right| = \frac{|\lambda|}{3}$$

Now $\left(\sqrt{\frac{\lambda^2}{4} + 4}\right)^2 - \left(\frac{|\lambda|}{3}\right)^2 = (3)^2$

$\therefore \lambda = \pm 6$

Since centre lies on positive z-axis

$\therefore \lambda = -6$

and equation of the sphere is $x^2 + y^2 + z^2 - 6z - 4 = 0$

Q.20 (B)

Let the point P(x, y, z)

Asking minimum value of OP^2

$\Rightarrow \perp^r$ distance of origin from plane

$$d = \left|\frac{P}{\sqrt{a^2 + b^2 + c^2}}\right| \Rightarrow d^2 = \frac{P^2}{\Sigma a^2}$$

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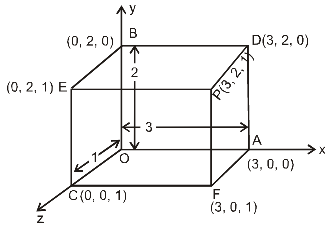
MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A, B, C)

dr's of $\overrightarrow{OP}$ are 3, 2, 1

dr's of $\overrightarrow{FB}$ are – 3, 2, – 1

dr's of $\overrightarrow{AE}$ are – 3, 2, 1



dr's of $\overrightarrow{CD}$ are 3, 2, – 1

$$\cos\theta_1 = \left|\frac{-9+4-1}{14}\right| = \frac{3}{7}, \cos\theta_2 = \left|\frac{-9+4+1}{14}\right| = \frac{2}{7}$$

$$\cos\theta_3 = \left|\frac{9+4-1}{14}\right| = \frac{6}{7}$$

So angles are $\cos^{-1}\frac{2}{7}$, $\cos^{-1}\frac{3}{7}$, $\cos^{-1}\frac{6}{7}$

Q.2 (B, D)

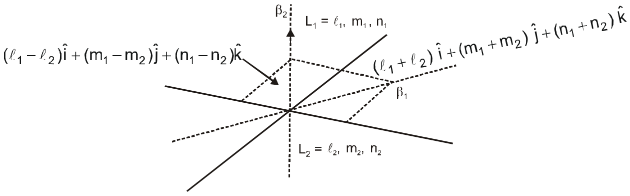
$\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$

Dc's of β_1 (bisector)

$$\frac{\ell_1 + \ell_2}{\sqrt{(\ell_1 + \ell_2)^2 + (m_1 + m_2)^2 + (n_1 + n_2)^2}}$$

$$= \frac{\ell_1 + \ell_2}{\sqrt{2 + 2(\ell_1\ell_2 + m_1m_2 + n_1n_2)}}$$

$$= \frac{\ell_1 + \ell_2}{\sqrt{2 + 2\cos\theta}} = \frac{\ell_1 + \ell_2}{2\cos\theta/2}$$



Similarly $\frac{m_1 + m_2}{2\cos\theta/2}$, $\frac{n_1 + n_2}{2\cos\theta/2}$

Similarly Dc's for bisector β_2

$$\frac{\ell_1 - \ell_2}{2\sin\frac{\theta}{2}}, \frac{m_1 - m_2}{2\sin\frac{\theta}{2}}, \frac{n_1 - n_2}{2\sin\frac{\theta}{2}}$$

Q.3 (B,C)

$$\hat{n} = \pm\left(\frac{-3,2,6}{7}\right) = \pm\left(\frac{-3}{7}, \frac{2}{7}, \frac{6}{7}\right)$$

$$-\frac{3x}{7} + \frac{2y}{7} + \frac{6z}{7} = 7$$

$– 3x + 2y + 6z – 49 = 0$

and $\frac{3x}{7} - \frac{2y}{7} - \frac{6z}{7} = 7$

$3x – 2y – 6z – 49 = 0$

Q.4 (A,D)

$$\frac{x-1}{2} = \frac{y+3}{1} = \frac{z+1}{-2}$$

Direction of line $\vec{b} = (2, –1, –2)$

(A) Normal of plane $\vec{n} = (2, 2, 1)$ $\vec{b} \cdot \vec{n} = 4 – 2 – 2 = 0$

(B) $\vec{b} \cdot \vec{n} = 2 – 2 + 4 = 4$

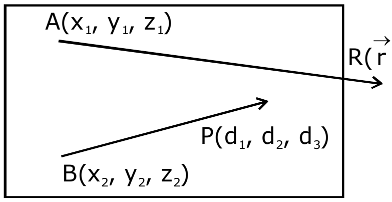
(C) $\vec{b} \cdot \vec{n} = 4 + 2 – 2 = 4$

(D) $\vec{b} \cdot \vec{n} = 2 + 2 – 4 = 0$

Q.5 (A,B)

$[\vec{AR}\ \vec{AB}\ \vec{P}] = 0$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$



or $$\begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

Q.6 (A,B)

$x + y + z – 1 = 0$ & $4x + y – 2z + 2 = 0$

put $z = 0$

$x + y = 1$

$4x + y = – 2 > x = –1, y = 2$

Point (–1, 2, 0)

Direction = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 4 & 1 & -2 \end{vmatrix}$

$= \hat{i}\,(–2 – 1) – \hat{j}\,(–2 – 4) + \hat{k}\,(1 – 4)$

$= –3\hat{i} + 6\hat{j} – 3\hat{k} = –3\,(1, –2, 1)$

Equation of line in symmetrical form

$\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-0}{1}$ (C) will also satisfy

Q.7 (A, B, C, D)

$\frac{x}{2} = \frac{y}{-3} = \frac{z}{6} = r$ (Let) …(1)

Let N(2r, –3r, 6r)

$\because$ PN is perpendicular to (1)

$\Rightarrow$ $(2r – 1) \times 2 + (–3r – 2) \times (– 3) + (6r – 5) \times 6 = 0$

$4r – 2 + 9r + 6 + 36r – 30 = 0$

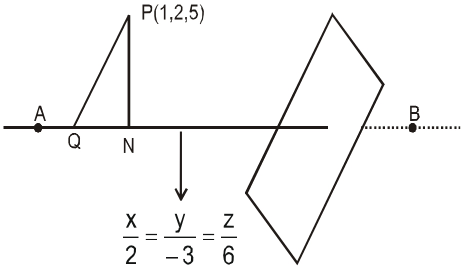
$49r – 26 = 0$

$r = \frac{26}{49}$

$\therefore$ $N\left(\frac{52}{49}, -\frac{78}{49}, \frac{156}{49}\right)$

$\because$ Equation of line PN is

$$\frac{x-1}{\frac{52}{49}-1} = \frac{y-2}{\frac{-78}{49}-2} = \frac{z-5}{\frac{156}{49}-5}$$



$$\frac{x-1}{3} = \frac{y-2}{-176} = \frac{z-5}{-89}$$

PQ is parallel to the plane $3x + 4y + 5z = 0$

Let co-ordinate of Q are $(2\ell, –3\ell, 6\ell)$

Dr's of PQ are $2\ell – 1, –3\ell – 2, 6\ell – 5$

$\therefore$ $3(2\ell –1) + 4(–3\ell –2) + 5\,(6\ell –5) = 0$

$\Rightarrow$ $\ell = 3/2$

Q (3, –9/2, 9)

Equation of the line PQ is

$$\frac{x-1}{2} = \frac{y-2}{-13/2} = \frac{z-5}{4}$$

or $\frac{x-1}{4} = \frac{y-2}{-13} = \frac{z-5}{8}$

$\frac{x}{2} = \frac{y}{-3} = \frac{z}{6} = r$ …(1)

N(2r, –3r, 6r)

∵ PN, (1)

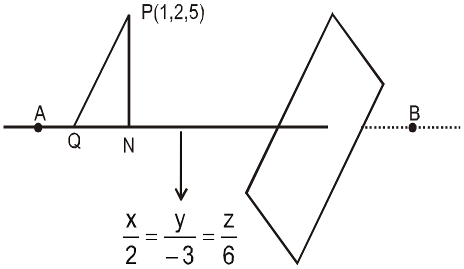
$\Rightarrow$ (2r – 1) × 2 + (–3r – 2) × (–3) + (6r – 5) × 6 = 0

4r – 2 + 9r + 6 + 36r – 30 = 0

49r – 26 = 0

$r = \dfrac{26}{49}$

$\therefore N\left(\dfrac{52}{49}, -\dfrac{78}{49}, \dfrac{156}{49}\right)$



$$\frac{x-1}{\frac{52}{49}-1} = \frac{y-2}{\frac{-78}{49}-2} = \frac{z-5}{\frac{156}{49}-5}$$

$$\frac{x-1}{3} = \frac{y-2}{-176} = \frac{z-5}{-89}$$

Q.8 (A, B)

$$\left.\begin{array}{r} 2x-3y-7z=0 \\ 3x-14y-13z=0 \\ 8x-31y-33z=0 \end{array}\right\}$$ obviously all the three planes pass through origin

$$D = \begin{vmatrix} 2 & -3 & -7 \\ 3 & -14 & -13 \\ 8 & -31 & -33 \end{vmatrix}$$

= 2(462 – 403) + 3(– 99 + 104) – 7(– 93 + 112)

= 118 + 15 – 133 = 0

From the theory of system of equations

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow$ System of equations has infinite solutions

$\therefore$ hence three planes intersect in a common line

Q.9 (A, C, D)

Let $\vec{a}, \vec{b}, \vec{c}$ are non coplanar

then $\vec{n} = x\vec{a} + y\vec{b} + z\vec{c}$

$\vec{n}.\vec{a} = 0 \Rightarrow x\vec{a}.\vec{a} + y\vec{a}.\vec{b} + z\vec{a}.\vec{c} = 0$

$\vec{n}.\vec{b} = 0 \Rightarrow x\vec{b}.\vec{a} + y\vec{b}.\vec{b} + z\vec{b}.\vec{c} = 0$

$\vec{n}.\vec{c} = 0 \Rightarrow x\vec{c}.\vec{a} + y\vec{c}.\vec{b} + z\vec{c}.\vec{c} = 0$

Since x, y, z are not all zero simultaneously

$$\Rightarrow \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = 0 \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]^2 = 0$$

(B) $\cos^2(30^\circ) + \cos^2(45^\circ) + \cos^2\gamma = 1$

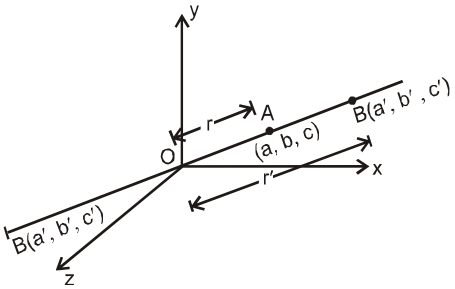
$\Rightarrow \dfrac{3}{4} + \dfrac{1}{2} + \cos^2\gamma = 1$ which is not possible

(C) Point is (3,4, λ) since only z co-ordinate changes. It lies on a line parallel to z axis and its distance from z axis = $\sqrt{9+16} = 5$

(D) Obivious

Q.10 (A, B)

Dr's of OA line ≡ a, b, c



Dr's of OB line ≡ a′, b′, c′

$\therefore \dfrac{a'}{a} = \dfrac{b'}{b} = \dfrac{c'}{c}$

Let Dc's of AB be ℓ, m, n

a = ℓr, b = mr, c = nr

a′ = ℓr′ b′ = mr′ c′ = nr′

aa′ + bb′ + cc′ = rr′($\ell^2 + m^2 + n^2$)

aa′ + bb′ + cc′ = rr′

If point B lies opposite side of origin as A then

aa′ + bb′ + cc′ = – rr′ ($\ell^2 + m^2 + n^2$)

$aa' + bb' + cc' = -rr'$

$\therefore aa' + bb' + cc' = \pm rr'$

Q.11 (A, D)

$\because \vec{a}.\vec{b} = 0$ and (तथा) $\vec{a}.\vec{c} = 0$

$|\vec{b}| = |\vec{c}| = |\vec{b} - \vec{c}| = 4\sqrt{2}$

$|\vec{b} - \vec{c}|^2 = (4\sqrt{2})^2$

$|\vec{b}|^2 + |\vec{c}|^2 - 2\vec{b}.\vec{c} = 32$

$32 + 32 - 2\vec{b}.\vec{c} = 32$

$\vec{b}.\vec{c} = 16$

$\because \ |\vec{a}| = 2$

$$\overrightarrow{AP} = \frac{\vec{b}}{2} - \vec{a} = \frac{\vec{b} - 2\vec{a}}{2}$$

$$\because \quad \overrightarrow{OQ} = \frac{\vec{b} + \vec{c}}{2}$$

$$\cos\theta = \pm \frac{\overrightarrow{AP}.\overrightarrow{OQ}}{\left|\overrightarrow{AP}\right|.\left|\overrightarrow{OQ}\right|} \qquad \ldots(1)$$

$$\because \quad \left|\overrightarrow{AP}\right| = \left|\frac{\vec{b} - 2\vec{a}}{2}\right|$$

$$= \frac{1}{2}\sqrt{|\vec{b}|^2 + 4|\vec{a}|^2 - 4(\vec{a}\,.\,\vec{b})} = \frac{1}{2}\sqrt{32 + 16} = \frac{1}{2}$$

$\sqrt{48} = 2\sqrt{3}$

$$\left|\overrightarrow{OQ}\right| = \frac{1}{2}\sqrt{|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}.\vec{c}} = \frac{1}{2}\sqrt{32 + 32 + 32}$$

$$= \frac{1}{2}\sqrt{96} = 2\sqrt{6}$$

$$\because \ \overrightarrow{AP}.\overrightarrow{OQ} = \frac{1}{4}\left(|b|^2 + \vec{b}.\,\vec{c} - 2\vec{a}.\vec{b} - 2\vec{a}.\vec{c}\right)$$

$$= \frac{1}{4}(32 + 16) = 12$$

$$\cos\theta = \pm\frac{12}{2\sqrt{3}\ 2\sqrt{6}} = \pm\frac{3}{\sqrt{3}\sqrt{6}} = \pm\frac{1}{\sqrt{2}}$$

Q.12 (B,C)

Let a point Q $(3\lambda + 15, 8\lambda + 2, -5\lambda + 6)$

PQ = $(2\lambda + 10, 8\lambda - 5, -5\lambda + 3)$

$3(3\lambda + 10) + 8(8\lambda - 5) - 5(-5\lambda + 3) = 0$

$9\lambda + 30 + 64\lambda - 40 + 25\lambda - 15 = 0$

$98\lambda = 35$

$\lambda = \frac{35}{98} \Rightarrow$ PQ = 14 (B)

and plane equation $9x - 4y - 14 = 0$

Q.13 (A, B, C)

Dr's of the line

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 4 & 1 & -2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(-2-4) + \hat{k}(1-4)$$

$= -3\hat{i} + 6\hat{j} - 3\hat{k}$

$\therefore$ Dr's are – 1, 2, – 1 or
1, – 2, 1

$x + y + z - 1 = 0$

$4x + y - 2z + 2 = 0$

Put z = 0

$x + y = 1$

$4x + y = -2$

$-3x = 3$

$x = -1, \ y = 2, z = 0$

$$\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-0}{1}$$

put z = 1

$$\left.\begin{array}{l} x + y = 0 \\ 4x + y = 0 \end{array}\right\} x = y = 0 \ \ \& \ z = 1$$

$$\therefore \ \frac{x}{1} = \frac{y}{-2} = \frac{z-1}{1}$$

put y = 1

$x + z = 0$

$4x - 2z = -3$

$2x + 2z = 0$

$\Rightarrow x = -\frac{1}{2}, z = \frac{1}{2}, y = 1$

$$\therefore \ \frac{x + \frac{1}{2}}{1} = \frac{y-1}{-2} = \frac{z - \frac{1}{2}}{1}$$

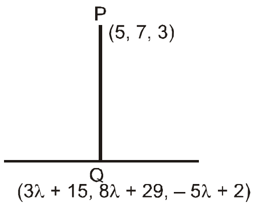
Q.14 (B, C)

d.r's of line are 3, 8, – 5

d.r's of PQ are $3\lambda + 10, 8\lambda + 22, -5\lambda + 2$

$\therefore$ both are perpendidcular

$\therefore (3\lambda + 10)3 + (8\lambda + 22)\,8 + (-5\lambda + 2)\,(-5) = 0$



i.e. $\lambda = -2$

$\therefore$ foot is (9, 13, 15), PQ = 14

Since (5, 7, 3), (9, 13, 15) lies on the plane

$9x - 4y - z - 14 = 0$

and $3 \times 9 + 8(-4) + (-5)(-1) = 0$

$\therefore$ equation of the required plane is

$9x - 4y - z - 14 = 0$

Comprehension # 1

Q.15 (C)

Q.16 (B)

Q.17 (A)

(15 to 17)

Let the co-ordinates of A be $(3\lambda + 3, 8 - \lambda, \lambda + 3)$ and the co-ordinates of B be $(-3\mu - 3, 2\mu - 7, 4\mu + 6)$.

Then direction ratios of AB are $3\lambda + 3\mu + 6$, $-\lambda - 2\mu + 15$, $\lambda - 4\mu - 3$

$AB \perp L_1$ so $3(3\lambda + 3\mu + 6) - (-\lambda - 2\mu + 15) + (\lambda - 4\mu - 3) = 0$

i.e. $11\lambda + 7\mu = 0$

and $AB \perp L_2$ so $-3(3\lambda + 3\mu + 6) + 2(-\lambda - 2\mu + 15) + 4(\lambda - 4\mu - 3) = 0$

i.e. $-7\lambda - 29\mu = 0$

$\Rightarrow \lambda = \mu = 0$

so the point A is (3, 8, 3) and the point B is (–3, –7, 6)

$\therefore AB = \sqrt{36 + 225 + 9} = \sqrt{270} = 3\sqrt{30}$

Comprehension # 2

Q.18 (B)

The equation of any plane through the intersection of P_1 and P_2 is

$$P_1 + \lambda P_2 = 0$$

$\Rightarrow$ (2x–y+z–2)+λ (x+2y–z–3)=0 ...(i)

Since, it passes through (3, 2, 1), then

$(6 - 2 + 1 - 2) + \lambda(3 + 4 - 1) = 0$

$\therefore \lambda = -1$

From Eq. (i),

$x - 3y + 2z + 1 = 0$

which is the required plane.

Q.19 (C)

The equation of any plane through (–1, 3, 2) is

$a(x = 1) + b(y - 3) + c(z - 2) = 0$ ...(ii)

If this plane (ii) is perpendicular to P_1, then

$2a - b + c = 0$ ...(iii)

From Eqs. (ii) and (iii), we get

$$\frac{a}{-1} = \frac{b}{3} = \frac{c}{5}$$

Substituting these proportionate values of a, b, c in Eq. (ii),

we get the required equation as

$-(x + 1) + 3(y - 3) + 5(z - 2) = 0$

or $x - 3y - 5z + 20 = 0$

Q.20 (A)

The given planes can be written as

$-2x + y - z + 2 = 0$ and $-x - 2y + z + 3 = 0$

Here, $(-2)(-1) + (1)(-2) + (-1)(1) = -1 < 0$

Equation of bisectors

$$\frac{(-2x + y - z + 2)}{\sqrt{(4 + 1 + 1)}} = \pm \frac{(-x - 2y + z + 3)}{\sqrt{1 + 4 + 1}}$$

$\therefore$ Acute angle bisector is

$(-2x + y - z + 2) = (-x - 2y + z + 3)$

$\Rightarrow x - 3y + 2s + 1 = 0$

Comprehension # 3

Q.21 (B)

Equation of the second plane is $-x + 2y - 3z + 5 = 0$

$2(-1) + 3 \cdot 2 + (-4)(-3) > 0$

$\therefore$ O lies in obtuse angle.

$(2 \times 1 + 3(-2) - 4 \times 3 + 7)(-1 + 2(-2) - 3 \times 3 + 5)$

$= (2 - 6 - 12 + 7)(-1 - 4 - 9 + 5) > 0$

$\therefore$ P lies in obtuse angle.

Q.22 (C)

$1 \times 2 + 2 \times 1 - 3 \times 3 < 0$

$\therefore$ O lies in acute angle.

Also $(2 + 2(-1) - 3(2) + 5)(2 \times 2 - 1 + 3 \times 2 + 1)$

$= (-1)(10) < 0$

$\therefore$ P lies in obtuse angle.

Q.23 (A)

$1 - 4 - 9 < 0$

$\therefore$ O lies in acute angle.

Further

$(1 + 4 - 6 + 2)(1 - 4 + 6 + 7) > 0$

$\therefore$ The point P lies in acute angle.

Q.24 $(A \to (r), (B) \to (s), (C) \to (q), (D) \to (p)$

(A) $y = 0, \ \dfrac{3\lambda - 1}{\lambda + 1} = 0 \quad \Rightarrow \lambda = \dfrac{1}{3}$

(B) $x + 3y - 4z = -6$

$$\frac{x}{-6} + \frac{y}{-2} + \frac{z}{3/2} = 0$$

Algebraic sum of intercept $\quad -6 - 2 + \dfrac{3}{2} = -\dfrac{13}{2}$

(C) $\cos\theta = \left|\dfrac{6 + 4 - 10}{5\sqrt{2}.3}\right| = 0$

(D) Let $A(2\lambda + 1, 4\lambda + 3, 3\lambda + 2)$

$(2\lambda + 1 - 3) \cdot 3 + (4\lambda + 3 - 8) \cdot 2 + (3\lambda + 2 - 2)(-2) = 0$

$6\lambda - 6 + 8\lambda - 10 - 6\lambda = 0$

$\lambda = 2$

$\therefore A(5, 11, 8)$

$\therefore AP = \sqrt{(2)^2 + (3)^2 + (6)^2} = 7$

Q.25 (A) - R; (B) Q ; (C) - S, P

$\because (2)(3) + (-1)(-2) + (2)(6) = 20 > 0$

$\therefore$ Bisectors are $\dfrac{(2x - y + 2z + 3)}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}$

$$= \pm \frac{(3x-2y+6z+8)}{\sqrt{(3)^2+(-2)^2+(6)^2}}$$ or 7(2x–y+2s+3)

= ± 3(3x – 2y + 6z + 8)

∴ Acute angle bisector is

7(2x – y + 2z + 3) = – 3(3x – 2y + 6z + 8)

⇒ 23x – 13y + 32z + 45 = 0 and Obtuse angle bisector is

7(2x – y + 2z + 3) = 3(3x – 2y + 6z + 8)

⇒ 5x – y – 4z – 3 = 0

∴ A : 23x – 13y + 32z + 45 = 0

and O : 5x – y – 4z – 3 = 0

(B) The Give planes can be written as

– x + 2y – 2z + 3 = 0 and 2x – 3y + 6z + 6z + 8 = 0

∵ (– 1) (2) + (2) (– 3) + (– 2) (6)

= – 2 – 6 – 12 = – 20 < 0

∴ Bisectors are,

$$\frac{(-x+2y-2z+3)}{\sqrt{(-1)^2+(2)^2+(-2)^2}} = \pm\frac{(2x-3y+6z+8)}{\sqrt{(2)^2+(-3)^2+(6)^2}}$$

⇒ 7 (– x+2y–2z+3)=± 3(2x – 3y + 6z + 8)

∴ Acuts angle bisector is 7 (– x + 2y – 2z + 8) = 3(2x – 3y + 6z + 8)

⇒ 13x – 23y + 32z +3 = 0 and obtuse bisector is

7 (– x+2y–2z+3) = – 3(2x – 3y + 6z + 8)

⇒ x – 5y – 4z – 45 = 0

⇒ ∴ A : 13x – 23y + 32z + 3 = 0

(C) The given planes can be written as

2x + y – 2z + 3=0 and – 6x – 2y+2z +8=0

∵ (2) (– 6) + (1) (– 2) + (– 2) (3)=–20<0

∴ Bisectors are $$\frac{(2x+y-2z+3)}{\sqrt{\{(2)^2+(1)^2+(-2)^2\}}}$$

$$= \pm\frac{(-6x-2y+3z+8)}{\sqrt{\{(-6)^2+(-2)^2+(3)^2\}}}$$

⇒ 7(2x + y – 2z + 3)=± 3(–6x–2y+ 3z+8)

∴ Acute angle bisector is 7(2x + y – 2x + 3) = 3(–6x – 2y + 3z + 8)

⇒ 32x + 13y – 23z – 3 = 0 and obtuse bisector is 7 (2x + y – 2x + 3)

= – 3(– 6x – 2y + 3z + 8)

⇒ 4x – y + 5z –45 = 0

∴ A : 32x + 13y –23z – 3 = 0 and

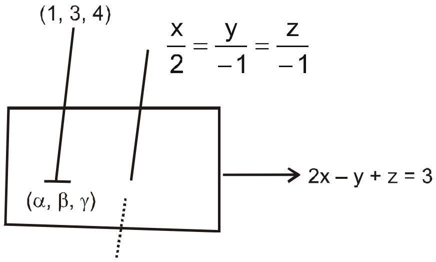
O : 4x – y + 5z – 45 = 0

Q.26 (A) → (p), (B) → (q), (C) → (r), (D) → (s)

(A) α – 1 = 2λ ⇒ α = 2λ + 1

β – 3 = – λ ⇒ β = – λ + 3

γ – 4 = – λ ⇒ γ = – λ + 4



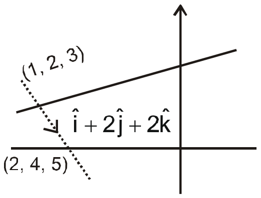
2(2λ + 1) – (– λ + 3) + (– λ + 4) = 3

$4\lambda = 0 \Rightarrow \lambda = 0$

$\alpha = 1,\ \beta = 3,\ \gamma = 4$

∴ distance = 0 (∵ given point lies on the plane)

(B) Common normal



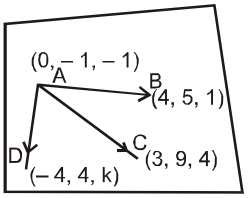
$$\begin{vmatrix} \hat{i} & \hat{i} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \hat{i}(15-16) - \hat{i}(10-12) + \hat{k}(8-9)$$

$$= -\hat{i}+2\hat{j}-\hat{k}$$

SD = projection of $+\hat{i}+2\hat{j}+2\hat{k}$ on

$$-\hat{i}+2\hat{j}-\hat{k} = \left|\frac{-1+4-2}{\sqrt{1+4+1}}\right| = \frac{1}{\sqrt{6}}$$

(C) $$\begin{vmatrix} -4 & 5 & k+1 \\ 3 & 10 & 5 \\ 4 & 6 & 2 \end{vmatrix} = 0$$



$$-4(20-30) - 5(6-20) + (k+1)(18-40) = 0$$

40 + 70 – 22 (k + 1) = 0 ⇒ k = 4

(D) Vertices of the tetrahedron are (0, 0, 0), (6, 0, 0), (0, – 4, 0), (0, 0, 3)

∴ Volume = $$\frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ 6 & 0 & 0 & 1 \\ 0 & -4 & 0 & 1 \\ 0 & 0 & 3 & 1 \end{vmatrix} = 12$$

NUMERCIAL VALUE BASED

Q.1 [6]

$\frac{x-2}{a}=\frac{y+1}{b}=\frac{z+1}{c}$ $\Rightarrow 4a+b+c=0$ ...(i)

$2x+y=0=x-y+z$ $\Rightarrow \begin{vmatrix} \hat{i} & \hat{i} & \hat{k} \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}(1-0)-\hat{j}(2-0)+\hat{k}(-2-1) = \hat{i}-2\hat{j}-3\hat{k}$

$a-2b-3c=0$ ... (ii)

From (i) & (ii)

$4a+b+c=0$ $\Rightarrow a-2b-3c=0$

$\Rightarrow \frac{a}{-3+2}=\frac{b}{1+12}=\frac{c}{-8-1} \Rightarrow \frac{a}{-1}=\frac{b}{13}=\frac{c}{-9}$

$\therefore$ equation of the line

$\frac{x-2}{-1}=\frac{y+1}{13}=\frac{z+1}{-9}$ $\Rightarrow \frac{3-2}{-1}=\frac{\alpha+1}{13}=\frac{\beta+1}{-9}$

$\Rightarrow \alpha=-14$ and $\beta=8 \Rightarrow |\alpha+\beta|=6.$

Q.2 [4]

$L_1: \frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}=r$; $L_2: 3x-2y+z+5=0$

$=2x+3y+4z-k$

Any point on the first line is $(3r-4, 5r-6, -2r+1)$

As lines are coplanar therefore this point must lie on both the planes representing the second line

$3(3r-4)-2(5r-6)+(-2r+1)+5=0$

$\Rightarrow$ $r=2$

and $2(3r-4)+3(5r-6)+4(-2r+1)-k=0$

$\Rightarrow k=4$

Q.3 [32]

Since $3(2)+4(-3)+6(1)=0$ and $3(1)+4(2)+6(-3)+7=0$

$\therefore$ the line $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z+3}{1}$ lies in the plane $3x+4y+6z+7=0$.

In the new position again the line lies in the plane. Let the equation of the new position of the plane be $ax+by+cz=0$, then $2a-3b+c=0$ and $a+2b-3c=0$

$\therefore$ $\frac{a}{9-2}=\frac{b}{1+6}=\frac{c}{4+3}$ i.e. $a=b=c$

$\therefore$ equation of the required plane is $x+y+z=0$

Q.4 [27]

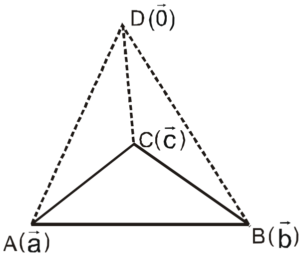
Since tetrahedron is regular AB = BC = AC = DC and angle between two adjcant side = $\pi/3$

consider planes ABD and DBC vector, normal to plane ABD is = $\vec{a}\times\vec{b}$

vector, normal to plane DBC is = $\vec{b}\times\vec{c}$ angle between these planes is angle between vectors $(\vec{a}\times\vec{b})$ & $(\vec{b}\times\vec{c})$

$$\Rightarrow \cos\theta=\frac{(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c})}{|\vec{a}\times\vec{b}||\vec{b}\times\vec{c}|}$$

$$=\frac{-\frac{1}{4}|\vec{b}|^2|\vec{a}||\vec{c}|}{\frac{3}{4}|\vec{a}||\vec{b}|^2|\vec{c}|}=-\frac{1}{3}$$



Since acute angle is required $\theta=\cos^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow \sec\theta=3 \Rightarrow \sec^3\theta=27$

Q.5 [17]

circum-radius $\equiv$ distance of circum centre from any of the vertex

$\equiv$ distance of $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ from vertex D$(\vec{0})$

[tetrahedron is regular]

Circumradius $=\frac{1}{4}|\vec{a}+\vec{b}+\vec{c}|=\frac{1}{4}$

$\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})}$

$$=\frac{1}{4}\sqrt{k^2+k^2+k^2+2\left(\frac{k^2}{2}+\frac{k^2}{2}+\frac{k^2}{2}\right)}$$

$$=\frac{1}{4}\sqrt{6k^2}=\sqrt{\frac{3}{8}}k$$

$\frac{r}{R}=\frac{1}{3}$ $\Rightarrow r=\frac{R}{3}=\frac{k}{\sqrt{24}}$ $\Rightarrow R=\sqrt{\frac{3}{8}}k$ &

$r=\frac{k}{\sqrt{24}}$ $\Rightarrow R^2+r^2=\frac{5}{12}k^2$

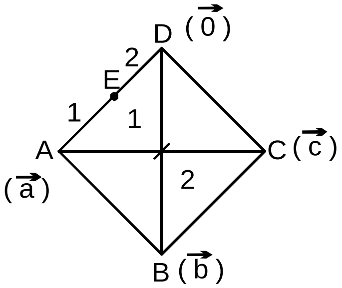
$\Rightarrow$ minimum value of p + q = 17

Q.6 [13]

$\sqrt{3^2+4^2+12^2}=13$

Q.7 [11]

$|\vec{a}| = |\vec{b}| = |\vec{c}| = |\vec{a}-\vec{b}| = |\vec{c}-\vec{b}| = |\vec{a}-\vec{c}| = a$



On solving we get

$\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{c} = \vec{a}\,.\,\vec{c} = \frac{a^2}{2} \Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = a$

$E = \left(\frac{2\vec{a}}{3}\right)$ & F = $\left(\frac{\vec{b}}{3}\right)$ area of ΔCEF = $\frac{1}{2}|\overrightarrow{CE}\times\overrightarrow{CF}|$

$= \frac{1}{2}\left|\left(\frac{2\vec{a}}{3}-\vec{c}\right)\times\left(\frac{\vec{b}}{3}-\vec{c}\right)\right|$

$= \frac{1}{2}\left|\frac{2}{9}(\vec{a}\times\vec{b}) + \frac{2}{3}(\vec{c}\times\vec{a}) + \frac{1}{3}(\vec{b}\times\vec{c})\right|$

$= \frac{1}{2}\left|\left(\frac{2+6+3}{9}\right)\vec{a}\times\vec{b}\right|$

$= \frac{1}{2}\cdot\frac{11}{9}|\vec{a}\times\vec{b}| \Rightarrow \frac{11}{18}\cdot|\vec{a}\times\vec{b}| \Rightarrow \frac{11}{18}\cdot|\vec{a}||\vec{b}|$ sin 60 $\Rightarrow \frac{11}{18}\,.\,a^2\,.\,\frac{\sqrt{3}}{2} = \frac{11\sqrt{3}}{36}a^2$

Q.8 [9]

Equation of the plane ABC will be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Now d = distance of the plane from origin O = $\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} \Rightarrow \frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

and m = OM = $\sqrt{a^2+b^2+c^2}$

So $\left(\frac{m}{d}\right)^2$ = $(a^2+b^2+c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)$ = 3 + $\left(\frac{a^2}{b^2}+\frac{b^2}{a^2}\right)+\left(\frac{b^2}{c^2}+\frac{c^2}{b^2}\right)+\left(\frac{c^2}{a^2}+\frac{a^2}{c^2}\right)$

$\Rightarrow \left(\frac{m}{d}\right)^2_{Min}$ = 3 + 6 = 9

By using A.M.–H. M. inequality, we get

$$\frac{a^2+b^2+c^2}{3} \geq \frac{3}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}} \Rightarrow (a^2+b^2+c^2)$$

$$\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right) \geq 9$$

Hence $(a^2+b^2+c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)_{minimum} = 9$

Q.9 [240]

Volume (V) = $\frac{1}{3}A_1\,h_1 \quad \Rightarrow h_1 = \frac{3V}{A_1}$

|||ly $h_2 = \frac{3V}{A_2}$, $h_3 = \frac{3V}{A_3}$ and $h_4 = \frac{3V}{A_4}$

So $(A_1 + A_2 + A_3 + A_4)(h_1 + h_2 + h_3 + h_4)$ = $(A_1 + A_2 + A_3 + A_4)\left(\frac{3V}{A_1}+\frac{3V}{A_2}+\frac{3V}{A_3}+\frac{3V}{A_4}\right)$

$= 3V(A_1 + A_2 + A_3 + A_4)$ $\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right)$

Now using A.M.-H.M inequality in A_1, A_2, A_3, A_4, we get

$$\frac{A_1+A_2+A_3+A_4}{4} \geq \frac{4}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right)}$$

$$\Rightarrow (A_1+A_2+A_3+A_4)\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right) \geq 16$$

Hence the minimum value of $(A_1+A_2+A_3+A_4)(h_1+h_2+h_3+h_4)$ = 3V (16)
= 48V = 48 × 5 = 240 **Ans.**

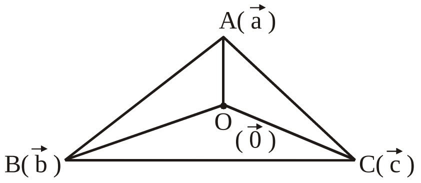
Q.10 [3]

Let position vector of A, B, C be $\vec{a}$, $\vec{b}$, $\vec{c}$ respectively.

$\therefore \quad 2\vec{a} + 5\vec{b} + 10\vec{c} = 0$ ……(i)

Taking cross product with $\vec{a}$ in (i)

$0 + 5\vec{a}\times\vec{b} + 10\vec{a}\times\vec{c} = 0$



$$\Rightarrow \quad \vec{a}\times\vec{b} = 2\vec{c}\times\vec{a}$$

Taking cross product with $\vec{c}$ in (i)

$$2\vec{a}\times\vec{c} + 5\vec{b}\times\vec{c} + 0 = 0$$

$$\Rightarrow \quad \vec{b}\times\vec{c} = \frac{2}{5}\vec{c}\times\vec{a}$$

$$\because \quad t = \frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta AOC}$$

$$= \frac{\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}{\frac{1}{2}\left|\vec{c}\times\vec{a}\right|}$$

$$= \frac{\left|2\vec{c}\times\vec{a}+\frac{2}{5}\vec{c}\times\vec{a}+\vec{c}\times\vec{a}\right|}{\left|\vec{c}\times\vec{a}\right|} = \frac{17}{5}$$

$$\therefore \quad |t| = 3$$

Q.11 [9]

$$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z-k}{3} = \lambda \Rightarrow (\lambda-2, 2\lambda+3, 3\lambda+k)$$

for A, $\lambda = 2$

$$A(0, 7, 6+k) \Rightarrow \text{for B} \quad \lambda = -\frac{k}{3}$$

$$\Rightarrow B\left(-2-\frac{k}{3}, 3-\frac{2k}{3}, 0\right)$$

$$\angle AOB = 90° \Rightarrow \overrightarrow{AO}\,.\,\overrightarrow{OB} = 0$$

$$\Rightarrow 7\left(-3+\frac{2k}{3}\right) = 0 \qquad \text{or } k = \frac{9}{2} \Rightarrow 2k = 9$$

Q.12 [34]

$P = xi + yj$; $\overrightarrow{AP} = (x-1)i + yj$

; $\overrightarrow{BP} = (x+1)i + yj$

$\overrightarrow{PA}\cdot\overrightarrow{PB} = x^2 - 1 + y^2$; $\overrightarrow{OA}\cdot\overrightarrow{OB} = -1$

Now $(\overrightarrow{PA}.\overrightarrow{PB}) + 3\,(\overrightarrow{OA}.\overrightarrow{OB}) = 0$

$\Rightarrow x^2 + y^2 - 4 = 0$ $\Rightarrow x^2 + y^2 = 4$

$$|\overrightarrow{PA}||\overrightarrow{PB}| = \sqrt{(x-1)^2+y^2}\ \sqrt{(x+1)^2+y^2}$$

$$= \sqrt{5-2x}\sqrt{5+2x}$$

$$|\overrightarrow{PA}||\overrightarrow{PB}| = \sqrt{25-4x^2}$$

Now from $x^2 + y^2 = 4$ put $x = 2\cos\theta$

$y = 2\sin\theta$

$$|\overrightarrow{PA}||\overrightarrow{PB}| = \sqrt{25-16\cos^2\theta}\ ;$$

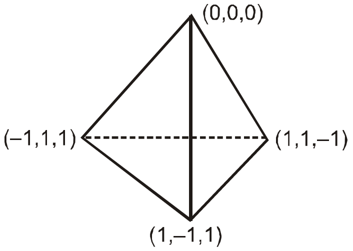
$$|\overrightarrow{PA}||\overrightarrow{PB}|_{\max} = \sqrt{25} = M$$

$$|\overrightarrow{PA}||\overrightarrow{PB}|_{\min} = \sqrt{9} = m\ ;\ M^2 + m^2 = 25 + 9 = 34$$

Q.13 [2]

The planes are

$y + z = 0$ ....(1)



$z + x = 0$ ....(2)

$x + y = 0$ ....(3)

$x + y + z = 1$ ....(4)

Solving above equations we get vertices of the tetrahedron as (0,0,0), (–1,1,1), (1,–1,1) and (1,1,–1)

$$\therefore \text{ Required volume} = \left|\frac{1}{6}\begin{vmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}\right|$$

$$= \left|\frac{1}{6}\begin{vmatrix}0 & 2 & 1\\ 2 & 0 & 1\\ 0 & 0 & -1\end{vmatrix}\right| = \frac{4}{6} = \frac{2}{3} \Rightarrow t = \frac{2}{3} \Rightarrow 3t = 2$$

Q.14 [4]

$$L_1 : \frac{x}{0} = \frac{y}{b} = \frac{z-c}{-c} = r$$

$$; L_2 : \frac{x}{a} = \frac{y}{0} = \frac{z+c}{c} = \ell$$

Dr's of AB are $-a\ell$, br, $-cr - c\ell + 2c$

$\Rightarrow$ AB is perpendicular to both the lines

$\therefore$ $0(-a\ell) + b.\,br + (-c)(-cr - c\ell + 2c) = 0$

$\Rightarrow (b^2 + c^2)\,r + c^2\ell = 2c^2$ .....(1)

and $a(-a\ell) + 0(br) + c(-cr - c\ell + 2c) = 0$

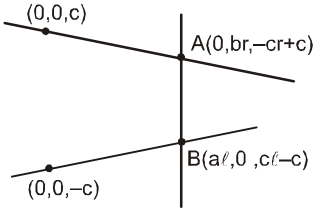
$\Rightarrow -(a^2 + c^2)\ell - c^2r + 2c^2 = 0$

$(a^2 + c^2)\ell + c^2r = 2c^2$ .....(2)

from (1) & (2)

$$\ell = \frac{2b^2c^2}{a^2b^2+b^2c^2+c^2a^2}, \quad r = \frac{2a^2c^2}{a^2b^2+b^2c^2+c^2a^2}$$

$$A\left(0, \frac{2a^2bc^2}{a^2b^2+b^2c^2+c^2a^2}, c\left(\frac{a^2b^2+b^2c^2-c^2a^2}{a^2b^2+b^2c^2+c^2a^2}\right)\right)$$



$$B\left(\frac{2ab^2c^2}{a^2b^2+b^2c^2+c^2a^2}, 0, c\left(\frac{b^2c^2-a^2b^2-c^2a^2}{a^2b^2+b^2c^2+c^2a^2}\right)\right)$$

$$d^2 = \frac{4a^2b^4c^4}{(a^2b^2+b^2c^2+c^2a^2)^2}$$

$$+\frac{4a^4b^2c^4}{(a^2b^2+b^2c^2+c^2a^2)^2} + \frac{4c^2(a^4b^4)}{(a^2b^2+b^2c^2+c^2a^2)^2}$$

$$\frac{4}{d^2} = \frac{(a^2b^2+b^2c^2+c^2a^2)^2}{a^2b^4c^4+a^4b^2c^4+a^4b^4c^2}$$

$$= \frac{a^2b^2+b^2c^2+c^2a^2}{a^2b^2c^2} \Rightarrow \frac{4}{d^2} = \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

JEE-MAIN PREVIOUS YEAR'S

Q.1 (2)

$l^2 + m^2 + n^2 = 1$

$\therefore\ 2n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore\ l^2 + m^2 = \frac{1}{2}\ \&\ l + m = \frac{1}{\sqrt{2}}$

$\Rightarrow\ \frac{1}{2} - 2\,lm = \frac{1}{2}$

$\Rightarrow\ lm = 0$ or $m = 0$

$\therefore\ l = 0, m = \frac{1}{\sqrt{2}}$ or $l = \frac{1}{\sqrt{2}}$

$\therefore\ <0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}>$ or $<\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}>$

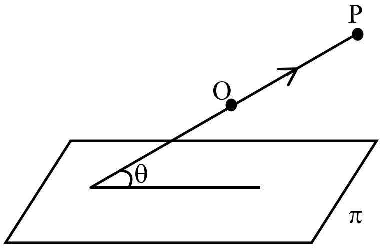
$\therefore\ \cos\alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2}$

$\therefore\ \sin^4\alpha + \cos^4\alpha = 1 - \frac{1}{2}\sin^2(2\alpha)$

$= 1 - \frac{1}{2}\cdot\frac{3}{4} = \frac{5}{8}$

Q.2 (1)

$$n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ -1 & 2 & 1 \end{vmatrix} = 5\hat{i} + \hat{j} + 3\hat{k}$$



$\therefore$ Required plane is : $5(x-2) + (y-1) + 3(z-3) = 0$

i.e. $5x + y + 3z = 20$

$|\overrightarrow{OP}| = \sqrt{4+1+0} = \sqrt{5}$

$|\overrightarrow{OP}| = 2\hat{i} - \hat{j}$

$$\sin\theta = \left|\frac{10-1}{\sqrt{5}\sqrt{25}+1+9}\right| = \frac{9}{5\sqrt{7}}$$

$\therefore$ Projection $= \sqrt{5} \times \cos\theta$

$$= \sqrt{5} \times \frac{1}{5}\sqrt{\frac{94}{7}} = \sqrt{\frac{94}{35}}.$$

Q.3 (1)

Plane passing through intersection of plane is

$\{\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = -1\} + \lambda\{\vec{r}\cdot(\hat{i}-2\hat{j}) + 2\} = 0$

Passes through $\hat{i} + 2\hat{k}$, we get

$(3-1) + \lambda(\lambda + 2) = 0 \quad \Rightarrow \lambda = -\frac{2}{3}$

Hence, equation of plane is $3\{\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) - 1\} - 2\{\vec{r}\cdot(\hat{i}-2\hat{j}) + 2\} = 0$

$\Rightarrow\ \vec{r}\cdot(\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

Q.4 (3)

$P_1 : x - 5y + 7z = 3$

$P_2 : 4x - 20y + 21z = 10$

$P_3 : x - 5y + 7z = 5$

P_1 and P_3 are parallel as dr's of normal are same

Q.5 [8]

$\overrightarrow{AB} = \hat{i} + 6\hat{j} - 2\hat{k}$

$\alpha = (\lambda - 4)\ \hat{i} + 4\hat{j} - \hat{k}$

$\overrightarrow{AB} .\alpha = 0$

$\lambda - 4 + 24 + 2 = 0 \qquad \Rightarrow \lambda = -22$

$E = 4 + 8 - 4 = 8$

Q.6 [4]

$$\frac{x-1}{4} = \frac{y-0}{-5} = \frac{z+1}{2} = \frac{-2(-6)}{16+25+4} = \frac{12}{45} = \frac{4}{15}$$

$$x–1 = \frac{16}{15} \Rightarrow x = \frac{31}{15}$$

$$y = -\frac{4}{3}$$

$$z+1 = \frac{8}{15} \Rightarrow z = -\frac{7}{15}$$

$$\alpha = \frac{31}{15}, \beta = -\frac{4}{3}, \gamma = -\frac{7}{15}$$

$$15(\alpha+\beta+\gamma) = \left(\frac{31}{15} - \frac{4}{3} - \frac{7}{15}\right) \times 15 = 4$$

Q.7 (1)

Let DR's of line are a,b,c

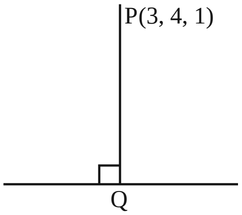
$\therefore$ a+2b+c=0

0.a+b+2c=0

$$\frac{a}{3} = \frac{b}{-2} = \frac{c}{1}$$

Points on the line is (–2, 4, 0)

$\therefore$ equation of lien is $\dfrac{x+2}{3} = \dfrac{y-4}{-2} = \dfrac{z}{1} = 1$



Points Q on the lien is (3λ–2, –2λ+4, λ)

DR's of PQ; 3λ–5, –2λ, λ–1

DR's of y lines are 3, –2, 1

Since PQ $\perp$ line $\Rightarrow$ 3(3λ–5)–2(–2λ)+1(λ–1)=0

$$\Rightarrow 14\lambda–16 \qquad \Rightarrow \lambda = \frac{8}{7}$$

$$\therefore\ Q\left(\frac{10}{7}, \frac{12}{7}, \frac{8}{7}\right)$$

Q.8 (1)

$$\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$$

$\Rightarrow x = \lambda + 3, y = 2\lambda\ 4, z = 2\lambda + 5$

Which lies on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$$\Rightarrow \lambda = \frac{5}{5} = 1$$

Hence, point of intersection is Q (4,6,7)

$\therefore$ Required distance = PQ

$= \sqrt{9 + 25 + 4}$

$= \sqrt{38}$

Q.9 (1)

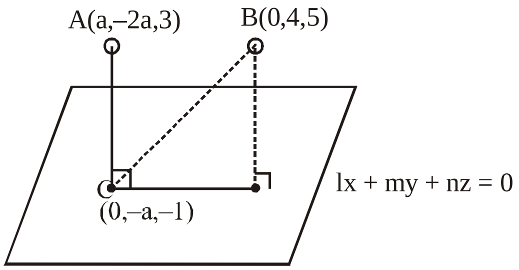
Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix}$

$= \hat{i} –11 – \hat{j} –17\hat{k}$

$\therefore \quad +11(x-1) + (y-2) + 17(z+3) = 0$

$11x + y + 17z + 38 = 0$

Q.10 (4)



C lies on plane $\Rightarrow$ -ma - n = 0 $\Rightarrow \dfrac{m}{n} = -\dfrac{1}{a}$ ....(1)

$\overrightarrow{CA} \parallel l\hat{i} + m\hat{j} + n\hat{k}$

$$\frac{a-0}{l} = \frac{-a}{m} = \frac{4}{n} \Rightarrow \frac{m}{n} = -\frac{a}{4} \qquad ....(2)$$

From (1) & (2)

$$-\frac{1}{a} = \frac{-a}{4} \Rightarrow a^2 = 4 \Rightarrow a = 2 \quad (\text{since } a > 0)$$

From (2) $\dfrac{m}{n} = \dfrac{-1}{2}$

Let m = -t $\Rightarrow$ n = 2t

$$\frac{2}{l} = \frac{-2}{-t} \Rightarrow l = t$$

So plane : t(x - y + 2z) = 0

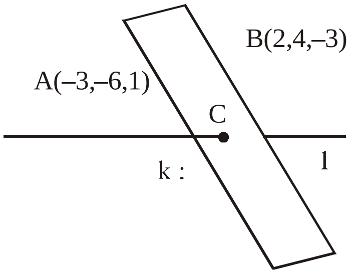
$$BD = \frac{6}{\sqrt{6}} = \sqrt{6} \qquad C \cong (0,-2,-1)$$

$$CD = \sqrt{BC^2 - BD^2}$$

$$= \sqrt{(0^2+6^2+6^2) - \left(\sqrt{6}\right)^2}$$

$$= \sqrt{66}$$

Q.11 (3)



Point C is

$$\left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$$

$$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$$

Plane $lx + my + nz = 0$

$l(-1) + m(2) + n(3) = 0$

$-l + 2m + 3n = 0$ ....(1)

It also satisfy point (1, –4, –2)

$l - 4m - 2n = 0$ ....(2)

Solving (1) and (2)

$2m + 3n = 4m + 2n$

$l\,n = 2m$

$l - 4m - 4m = 0$

$l = 8m$

$$\frac{l}{8} = \frac{m}{1} = \frac{n}{2}$$

$l : m : n = 8 : 1 : 2$

Plane is $8x + y + 2z = 0$

It will satisfy point C

$$8\left(\frac{2k-3}{k+1}\right) + \left(\frac{4k-6}{k+1}\right) + 2\left(\frac{-3k+1}{k+1}\right) = 0$$

$16k - 24 + 4k - 6 - 6k + 2 = 0$

$14k = 28 \quad \therefore \quad k = 2$

Q.12 [3]

Plane passing through (42, 0, 0), (0, 42, 0), (0, 0, 42)

From intercept from, equation of plane is

$x + y + z = 42$

$\Rightarrow (x - 11) + (y - 19) + (z - 12) = 0$

let $a = x - 11$, $b = y - 19$, $c = z - 12$

$a + b + c = 0$

Now, given expression is

$$3 + \frac{a}{b^2c^2} + \frac{b}{a^2c^2} + \frac{c}{a^2b^2} - \frac{42}{14abc}$$

$$3 + \frac{a^3 + b^3 + c^3 - 3abc}{a^2b^2c^2}$$

If $a + b + c = 0$

$\Rightarrow$ a3 + b3 + c3 = 3 abc

$\Rightarrow 3$

Q.13 (2)

(3,5,7) satisfy the line L_1 :

$$\frac{3-a}{\ell} = \frac{5-2}{3} = \frac{7-b}{4}$$

$$\frac{3-a}{\ell} = 1 \qquad \& \qquad \frac{7-b}{4} = 1$$

$a + l = 3$ ...(1) & $b = 3$ ...(2)

$\vec{V}_1 = <4,3,8> - <3,5,7>$

$\vec{V}_1 = <1,-2,1>$

$\vec{V}_2 = <\ell,3,4>$

$\vec{V}_1 \cdot \vec{V}_2 = 0 \Rightarrow \ell - 6 + 4 = 0 \Rightarrow \ell = 2$

$a + l = 3 \Rightarrow a = 1$

$$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

A = <1, 2, 3>

B = <2, 4, 5>

$\overline{AB} = <1,2,2>$

$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} \times \vec{q} = -\hat{i} + 2\hat{j} - \hat{k}$

$$\text{Shortest distance} = \left|\frac{\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right| = \frac{1}{\sqrt{6}}$$

Q.14 (2)

$\vec{r} \times \vec{a} = \vec{b} \times \vec{r} \quad \Rightarrow \quad \vec{r} \times (\vec{a} + \vec{b}) = 0$

$\vec{r} = \vec{\lambda}(\vec{a} + \vec{b}) \Rightarrow \vec{r} = \vec{\lambda}(\hat{i} + 2\hat{j} - 3\hat{k} + 2\hat{i} - 3\hat{j} + 5\hat{k})$

$\vec{r} = \vec{\lambda}(3\hat{i} - \hat{j} + 2\hat{k})$ ...(1)

$\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$

Put $\vec{r}$ from (1) $\alpha\lambda = 1$ ...(2)

$\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$

Put $\vec{r}$ from (1) $2\lambda\alpha-\lambda=1$ ...(3)

Solve (2) & (3)

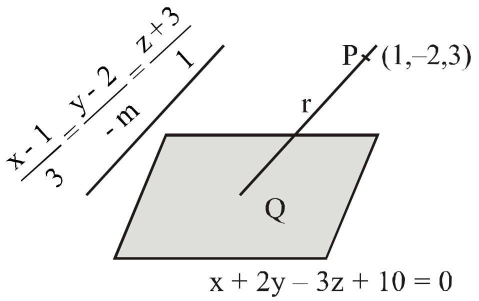
$\alpha = 1, \lambda = 1$

$\Rightarrow \vec{r}=3\hat{i}-\hat{j}+2\hat{k}$

$|\vec{r}|^2=14$ & $\alpha = 1$

$\alpha+|\vec{r}|^2=15$

Q.15 [2]



$$\text{DC of line} \equiv \left(\frac{3}{\sqrt{m^2+10}},\frac{-m}{\sqrt{m^2+10}},\frac{1}{\sqrt{m^2+10}}\right)$$

$$Q \equiv \left(+\frac{3r}{\sqrt{m^2+10}} - +\frac{-mr}{\sqrt{m^2+10}} \quad +\frac{r}{\sqrt{m^2+10}}\right)$$

Q lies on x + 2y − 3z + 10 = 0

$$1+\frac{3r}{\sqrt{m^2+10}}-4-\frac{2mr}{\sqrt{m^2+10}}-9-\frac{3r}{\sqrt{m^2+10}}+10 =0$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(3-2m-3)=2$$

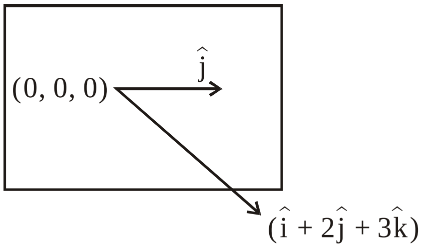
$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(-2m)=2$$

r2m2 = m2 + 10

$\frac{7}{2}m^2 = m^2 + 10 \Rightarrow \frac{5}{2}m^2 = 10 \Rightarrow m^2 = 4$

$|m| = 2$

Q.16 (4)



$\vec{n}=\hat{j}\times(\hat{i}+2\hat{j}+3\hat{k})$

$= -3\hat{i}+0\hat{j}+\hat{k}$

So,(-3) (x – 1) + 0 (y – 2) + (1) (z – 3) = 0

$\Rightarrow$ – 3x + z = 0

Option 4

Alternate :

Required plane is

$$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix}=0$$

$\Rightarrow$ 3x – z = 0

Q.17 [4]

Required plane is

$p1 + \lambda p_2 = (2 + 3\lambda)\, x – (7 + 5\lambda)\, y + (4 + 4\lambda)z – 3 + 11\lambda = 0$;

which is satisfied by (–2, 1, 3).

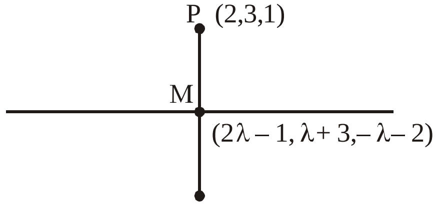
Hence, $\lambda = \frac{1}{6}$

Thus, plane is 15x – 47y + 28z – 7 = 0

So, 2a + b + c – 7 = 4

Q.18 (2)

Line $\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$



$\overrightarrow{PM}=(2\lambda-3, \lambda, -\lambda-3)$

$\overrightarrow{PM}\perp\left(2\hat{i}+\hat{j}-\hat{k}\right)$

$4\lambda – 6 + \lambda + \lambda + 3 = 0 \Rightarrow \lambda = \frac{1}{2}$

$\therefore M \equiv \left(\quad \frac{7}{2} \; \frac{-5}{2}\right)$

$\therefore$ Reflection (–2, 4, –6)

$$\text{Plane :} \begin{vmatrix} x-2 & y-1 & z+1 \\ & & \\ 4 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow$ (x – 2) (–10 + 3) – (y – 1) (15 – 4) + (z + 1) (–1) = 0

$\Rightarrow$ –7x + 14 – 11y + 11 – z – 1 = 0

$\Rightarrow$ 7x + 11y + z = 24

$\therefore$ $\alpha = 7, \beta = 11, \gamma = 1$

$\alpha + \beta + \gamma = 19$ **Option (2)**

Q.19 [0]

Let point P is (α, β, γ)

$$\left(\frac{\alpha+\beta+\gamma}{\sqrt{3}}\right)^2+\left(\frac{\ell\alpha-n\gamma}{\sqrt{\ell^2+n^2}}\right)^2+\left(\frac{\alpha-2\beta+\gamma}{\sqrt{6}}\right)^2=9$$

Locus is

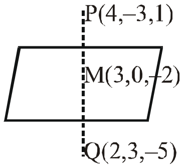
$$\frac{(x+y+z)^2}{3}+\frac{(\ell x-nz)^2}{\ell^2+n^2}+\frac{(x-2y+z)^2}{6}=9$$

$$x^2\left(\frac{1}{2}+\frac{\ell^2}{\ell^2+n^2}\right)+y^2+z^2\left(\frac{1}{2}+\frac{n^2}{\ell^2+n^2}\right)+2zx\left(\frac{1}{2}-\frac{\ell n}{\ell^2+n^2}\right)-9=0$$

Since its given that $x^2 + y^2 + z^2 = 9$

After solving l = n

Q.20 [28]



Plane is $1(x-3)-3(y-0)+3(z+2)=0$

$x-3y+3z+3=0$

$(a^2+b^2+c^2+d^2)_{min}=28$

Q.21 [4]

Let plane is $x-2y+2z+\lambda=0$

distance from (1,2,3) = 1

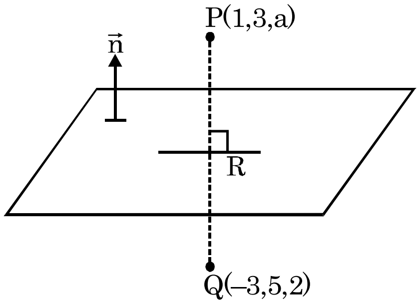
$\Rightarrow \frac{|\lambda+3|}{5}=1 \Rightarrow \lambda=0\ -6$

$\Rightarrow a=1, b=-2, c=2, d=-6$ or 0

$b-d=4$ or -2, $c-a=1$

$\Rightarrow k=4$ or -2

Q.22 [1]



plane = $2x-y+z=b$

$R\equiv\left(-1,4,\frac{a+2}{2}\right)\rightarrow$ on plane

$\therefore -2-4+\frac{a+2}{2}=b$

$\Rightarrow a+2=2b+12 \Rightarrow a=2b+10$ ...(i)

$<PQ> = <4, -2, a-2>$

$\therefore \frac{2}{4}=\frac{-1}{-2}=\frac{1}{a-2}$

$\Rightarrow a-2=2 \Rightarrow a=4, b=-3$

$\therefore |a+b|=1$

Q.23 [38]

Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0$

Now $(1, -1, \alpha)$ lies on it so

$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0 \Rightarrow 5\alpha+38=0 \Rightarrow |5\alpha|=38$

Q.24 (2)

Q.25 (1)

Q.26 (3)

Q.27 [1]

Q.28 (4)

Q.29 [3]

Q.30 (4)

Q.31 [7]

Q.32 [6]

Q.33 (1)

Q.34 (4)

Q.35 (2)

$P_1: x-2y-2z+1=0$

$P_2: 2x-3y-6z+1=0$

$$\left|\frac{x-2y-2z+1}{\sqrt{1+4+4}}\right|=\left|\frac{2x-3y-6z+1}{\sqrt{2^2+3^2+6^2}}\right|$$

$$\frac{x-2y-2z+1}{3}=\pm\frac{2x-3y-6z+1}{7}$$

Since $a_1a_2+b_1b_2+c_1c_2=20>0$

$\therefore$ Negative sign will give acute bisector

$7x-14y-14z+7=-[6x-9y-18z+3]$

$\Rightarrow 13x-23y-32z+10=0$

$\left(-2,0,-\frac{1}{2}\right)$ satisfy it $\therefore$ Ans (2)

Q.36 (3)

Q.37 (4)

Q.38 [7]

Q.39 (1)

Q.40 [61]

Q.41 (3)

Q.42 [96]

Q.43 (2)

Q.44 [26]

Q.45 (2)

Q.46 (4)

Q.47 (1)

Q.48 (4)

Q.49 [4]

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (A)

Equation of QR is $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1}$

Let $P \equiv (2+\lambda, 3+4\lambda, 5+\lambda)$

$10 + 5\lambda - 12 - 16\lambda - 5 - \lambda = 1$

$-7 - 12\lambda = 1$

$\Rightarrow \lambda = \frac{-2}{3}$

then $P \equiv \left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

Let $S = (2+\mu, 3+4\mu, 5+\mu)$

$\overrightarrow{TS} = (\mu)\hat{i} + (4\mu+2)\hat{j} + (\mu+1)\hat{k}$

$\overrightarrow{TS}\,.(\hat{i} + 4\hat{j} + \hat{k}) = 0$

$\mu + 16\mu + 8 + \mu + 1 = 0$

$\mu = -\frac{1}{2}$

$S = \left(\frac{3}{2}, 1, \frac{9}{2}\right)$

$$PS = \sqrt{\left(\frac{4}{3}-\frac{3}{2}\right)^2 + \frac{4}{9} + \left(\frac{13}{3}-\frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{1}{36} + \frac{4}{9} + \frac{1}{36}}$$

$$= \sqrt{\frac{1}{18} + \frac{4}{9}} = \sqrt{\frac{9}{18}} = \frac{1}{\sqrt{2}}$$

Q.2 (A)

Equation of required plane

$(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

$\Rightarrow (1+\lambda)x + (2-\lambda)y + (3+\lambda)z - (2+3\lambda) = 0$

distance from point $(3, 1, -1)$

$$= \left|\frac{3+3\lambda+2-\lambda-3-\lambda-2-3\lambda}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \left|\frac{-2\lambda}{\sqrt{3\lambda^2+4\lambda+14}}\right| = \frac{2}{\sqrt{3}}$$

$\Rightarrow 3\lambda^2 = 3\lambda^2 + 4\lambda + 14$

$\Rightarrow \lambda = -\frac{7}{2}$

equation of required plane

$5x - 11y + z - 17 = 0$

Q.3 (B,C)

For co-planer lines $[\vec{a}-\vec{c}\ \vec{b}\ \vec{d}] = 0$

$\vec{a} \equiv (1, -1, 0)$, $\vec{c} = (-1, -1, 0)$

$\vec{b} = 2\hat{i} + k\hat{j} + 2\hat{k}$ $\qquad$ $\vec{d} = 5\hat{i} + 2\hat{j} + k\hat{k}$

Now $\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \Rightarrow$ $\qquad$ $k = \pm 2$

$\vec{n}_1 = \vec{b}_1 \times \vec{d}_1 = 6\hat{j} - 6\hat{k}$ for $k = 2$

$\vec{n}_2 = \vec{b}_2 \times \vec{d}_2 = 14\hat{j} + 14\hat{k}$ for $k = -2$

so the equation of planes are $(\vec{r}-\vec{a}).\vec{n}_1 = 0 \Rightarrow y - z = -1$ ......(1)

$(\vec{r}-\vec{a}).\vec{n}_2 = 0 \Rightarrow y + z = -1$ ......(2)

so answer is (B,C)

Q.4 (D)

Any point on line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3} = \lambda$

Let any two points on this line are

A(– 2, – 1, 0), B (0, – 2, 3)   Put
($\lambda = 0, 1$)
Let foot of perpendicular from A(– 2, – 1, 0) on plane is (α, β, γ)

$$\Rightarrow \frac{\alpha+2}{1} = \frac{\beta+1}{1} = \frac{\beta-0}{1} = \mu \text{ (say)}$$

Also, $\alpha + \beta + \gamma = 3$
$\Rightarrow \mu - 2 + \mu - 1 + \mu = 3 \Rightarrow \mu = 2$
$\Rightarrow$ M(0, 1, 2)
Similarly foot of perpendicular from B(0, – 2, 3) on

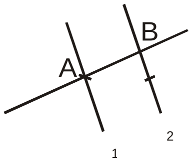
plane is N $\left(\frac{2}{3}, \frac{-4}{3}, \frac{11}{3}\right)$

So, equation of MN is $\dfrac{x-0}{\frac{2}{3}} = \dfrac{y-1}{\frac{-7}{3}} = \dfrac{z-2}{\frac{5}{3}}$.

Q.4 (B,D)
Let equation of line ℓ is

$$\ell : \frac{x-0}{a} = \frac{y-0}{b} = \frac{z-0}{c} = k$$

This line ℓ is perpendicular to given line ℓ_1 and ℓ_2.



Hence a + 2b + 2c = 0
2a + 2b + c = 0

$$\frac{a}{-2} = \frac{b}{3} = \frac{c}{-2}$$

Hence equation of ℓ is $\quad \dfrac{x}{-2} = \dfrac{y}{3} = \dfrac{z}{-2} = k_1,\ k_2$
$\downarrow \quad \downarrow$
for ℓ_1 for ℓ_2
Now A($-2k_1, 3k_1, -2k_1$)   B($-2k_2, 3k_2, -2k_2$)
Point A satisfied ℓ_1

$-2k_1\hat{i} + 3k_1\hat{j} - 2k_1\hat{k} = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$

$3 + t = -2k_1$ ....(1)
$-1 + 2t = 3k_1$ ....(2)
$4 + 2t = -2k_1$ ....(3)
(2) & (3) $-5 = 5k_1 \Rightarrow k_1 = -1 \Rightarrow$ A (2, –3, 2)
Let any point on ℓ_2 (3 + 2S, 3 + 2S, 2 + S)

Given $\sqrt{(1+2S)^2 + (6+2S)^2 + (S)^2} = \sqrt{17}$

$9S^2 + 28S + 37 = 17$
$9S^2 + 28S + 20 = 0$
$9S^2 + 18S + 10S + 20 = 0$
$9S(S + 2) + 10 (S + 2) = 0$
S = –2, –10/9
Hence (–1, –1, 0) , (7/9, 7/9, 8/9)

Q.5 (A, D)

$$\frac{x-5}{0} = \frac{-y}{\alpha-3} = \frac{z}{-2}$$

$$\frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$(5 - \alpha) ((3 - \alpha) (2 - \alpha) - 2) = 0$
$(\alpha^2 - 5\alpha + 6 - 2) = 0$
$(\alpha - 5)(\alpha^2 - 5\alpha + 4) = 0$
$\alpha = 1, 4, 5$

Q.6 (A)

$$L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$$

Normal of plane P : $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix}$

$= \hat{i}(-16) - \hat{j}(-42 - 6) + \hat{k}(32)$

$= -16\hat{i} + 48\hat{j} + 32\hat{k}$

$\Rightarrow \vec{n} = \hat{i} - 3\hat{j} - 2\hat{k}$
Point of intersection of L_1 and L_2
$2k_1 + 1 = k_2 + 4$
$-k_1 = k_2 - 3$
$1 = 3k_2 - 2$
$k_2 = 1$
Point of intersection (5, –2, –1)
Plane (x – 5) – 3 (y + 7) – 2(z + 1) = 0
x – 3y – 2z – 5 – 6 – 2 = 0
x – 3y – 2z = 13
$\Rightarrow$ a = 1, b = 3, c = –2, d = 13

Q.7 (C)
Line is

$$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1} = \alpha \quad ......(1)$$

Q(α, α, 1)
Direction ratio of PQ are
$\lambda - \alpha, \lambda - \alpha, \lambda - 1$
Since PQ is perpendicular to (1)
$\therefore \lambda - \alpha + \lambda - \alpha + 0 = 0$
$\lambda = \alpha$
$\therefore$ Direction ratio of PQ are
0, 0, $\lambda - 1$

Another line is $\frac{x-0}{-1} = \frac{y-0}{1} = \frac{z+1}{0} = \beta$ ......(2)

$\therefore R(-\beta, \beta, -1)$

$\therefore$ Direction ratio of PR are $\lambda + \beta, \lambda - \beta, \lambda + 1$

Since PQ is perpendicular to (ii)

$\therefore -\lambda - \beta + \lambda - \beta = 0$

$\beta = 0$

$\therefore$ R(0, 0, – 1) and Direction ratio of PQ are $\lambda, \lambda, \lambda + 1$

Since $PQ \perp PR$

$\therefore 0 + 0 + \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1 \Rightarrow$ B, C

For $\lambda = 1$ the point is on the line so it will be rejected.

$\Rightarrow \lambda = -1$.

Q.8 (B,D)

Let P_3 be $P_2 + \lambda P_1 = 0 \Rightarrow$ $x + \lambda y + z - 1 = 0$

Distance from (0, 1, 0) is 1

$$\therefore \frac{0+\lambda+0-1}{\sqrt{1+\lambda^2+1}} = \pm 1$$

$$\lambda = -\frac{1}{2}$$

$\therefore$ Equation of P_3 is $2x - y + 2z - 2 = 0$

Dist. from (α, β, γ) is 3

$$\therefore \left|\frac{2\alpha - \beta + 2\gamma - 2}{3}\right| = 2 \qquad \Rightarrow 2\alpha - \beta + 2\gamma = 2 \pm 6$$

$\therefore$ option (B, D) are correct.

Q.9 (A,B)

Let $\vec{v}$ be the vector along L

$$\text{then } \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \hat{i} - 3\hat{j} - 5\hat{k}$$

So any point on line L is $A(\lambda, -3\lambda, -5\lambda)$

Foot of perpendicular from A to P, is

$$\frac{h-\lambda}{1} = \frac{k-3\lambda}{2} = \frac{\ell+5\lambda}{-1} = -\frac{(\lambda - 6\lambda + 5\lambda + 1)}{1+4+1} = -\frac{1}{6}$$

$$h = \lambda - \frac{1}{6},\ k = -3\lambda - \frac{1}{3},\ \ell = -5\lambda + \frac{1}{6}$$

so foot is $\left(\lambda - \frac{1}{6}, -3\lambda - \frac{1}{3}, -5\lambda + \frac{1}{6}\right)$ So (A, B)

Q.10 (C)

Image of point (3, 1, 7) in plane $(x - y + z = 3)$ is P

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-7}{1} = -2\left(\frac{3-1+7-3}{1+1+1}\right) = -4$$

So P is (–1, 5, 3)

Equation of plane through P and containing the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is

$ax + by + cz = 0$

$a + 2b + c = 0$

$-a + 5b + 3c = 0$

$$\therefore \begin{vmatrix} x & y & z \\ 1 & 2 & 1 \\ -1 & 5 & 3 \end{vmatrix} = 0 \qquad \Rightarrow x - 4y + 7z = 0$$

Q.11 (A,C,D)

$$\because \pi r^2 = \frac{8\pi}{3} \qquad \Rightarrow r = \frac{2\sqrt{2}}{\sqrt{3}}$$

$$\text{Also } \frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2} = \lambda$$

$\Rightarrow s = 9\lambda$, “ $= rs$

$$\because \Delta^2 = s(s-x)(s-y)(s-z) \quad \Rightarrow \lambda = 1$$

$s = 9$ and sides are 5, 6 and 7.

$$“ = 9 \times \frac{2\sqrt{2}}{\sqrt{3}} = 6\sqrt{6}$$

$$\Delta = \frac{xyz}{4R} \qquad \Rightarrow R = \frac{35}{4\sqrt{6}}$$

$$r = 4R \sin\frac{X}{2} \sin\frac{Y}{2} \sin\frac{Z}{2}$$

$$\Rightarrow \sin\frac{X}{2} \sin\frac{Y}{2} \sin\frac{Z}{2} = \frac{r}{4R} = \frac{4}{35}$$

$$\sin^2\left(\frac{\pi}{2} - \frac{Z}{2}\right) = \frac{1}{2}(1 + \cos Z) \qquad \because \cos Z = \frac{1}{5}$$

Hence, (a, c, d)

Q.12 (D)

Let plane be

$a(x-1) + b(y-1) + c(z-1) = 0$

Now, direction ration of its normal

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = \hat{i}(-14) - \hat{j}(2) + \hat{k}(-15)$$

So, $-14(x-1) - 2(y-1) - 15(z-1) = 0$

$14x + 2y + 15z = 31$

Q.13 (C,D)

D.C. of line of intersection (a, b, c)

$\Rightarrow 2a + b - c = 0$

$a + 2b + c = 0$

$$\frac{a}{1+2}=\frac{b}{-1-2}=\frac{c}{4-1}$$

(B) $\frac{3x-4}{9}=\frac{1-3y}{9}=\frac{z}{3}$

$$\Rightarrow \frac{x-4/3}{3}=\frac{y-1/3}{-3}=\frac{z}{3}$$

$\Rightarrow$ lines are parallel.

(C) Acute angle between

P_1 and P_2 = $\cos^{-1}\left(\frac{2\times1+1\times2-1\times1}{\sqrt{6}\sqrt{6}}\right)$

$= \cos^{-1}\left(\frac{3}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^{o}$

(D) Plane is given by $(x-4)-(y-2)+(z+2)=0$

$\Rightarrow\ x-y+z=0$

Distance of (2, 1, 1) from plane = $\frac{2-1+1}{\sqrt{3}}=\frac{2}{\sqrt{3}}$

Q.14 [8]

Let $P(\alpha,\beta,\gamma)$

$Q(0,0,\gamma)$&

$R(\alpha,\beta,-\gamma)$

Now, $\overline{PQ}\,||\,\hat{i}+\hat{j}\Rightarrow(\alpha\hat{i}+\beta\hat{j})\,||\,(\hat{i}+\hat{j})$

$\Rightarrow\alpha=\beta$

Also, mid point of PQ lies on the plane

$\Rightarrow\frac{\alpha}{2}+\frac{\beta}{2}=3\Rightarrow\alpha+\beta=6\Rightarrow\alpha=3$

Now, distance of point P from X-axis is $\sqrt{\beta^2+\gamma^2}=5$

$\Rightarrow\beta^2+\gamma^2=25\Rightarrow\gamma^2=16$

as $\beta=\alpha=3$

as $\gamma=4$

Hence, PR = $2\gamma=8$

Q.15 (A,B,D)

Points on L_1 and L_2 are respectively A $(1-\lambda, 2\lambda, 2\lambda)$ and B$(2\mu, -\mu, 2\mu)$

So, $\overline{AB}$ = $(2\mu+\lambda-1)\,\hat{i}$ + $(-\mu-2\lambda)\,\hat{j}$ + $(2\mu-2\lambda)\,\hat{k}$

and vector along their shortest distance = $2\hat{i}+2\hat{j}-\hat{k}$.

Hence, $\frac{2\mu+\lambda-1}{2}=\frac{-\mu-2\lambda}{2}=\frac{2\mu-2\lambda}{-1}$

$\Rightarrow\lambda=\frac{1}{9}$ & $\mu=\frac{2}{9}$

Hence, A $\equiv\left(\frac{8}{9},\frac{2}{9},\frac{2}{9}\right)$ and B $\equiv\left(\frac{4}{9},-\frac{2}{9},\frac{4}{9}\right)$

$\Rightarrow$ Mid point of AB $\equiv\left(\frac{2}{3},0,\frac{1}{3}\right)$

Q.16 (3,4)

Let P$(\lambda, 0, 0)$, Q$(0, \mu, 1)$, R$(1,1,\nu)$ be points. L_1, L_2 and L_3 respectively

Since P, Q, R, are collinear, $\overrightarrow{PQ}$ iscollinear with $\overrightarrow{QR}$

Hence $\frac{-\lambda}{1}=\frac{\mu}{1-\mu}=\frac{1}{\nu-1}$

For every $\mu\in R-\{0,1\}$ there exist unique $\lambda, \nu\in R$

Q.17 (A,B)

Point of intersection of L_1 & L_2 is (1, 0, 1)

Line L passes through (1, 0, 1)

$\frac{1-\alpha}{\ell}=-\frac{1}{m}=\frac{1-\gamma}{-2}$ ...(1)

acute angle bisector of L_1 & L_2

$\vec{r}=\hat{i}+\hat{k}+\lambda\left(\frac{\hat{i}-\hat{j}+3\hat{k}-3\hat{i}-\hat{j}+\hat{k}}{\sqrt{11}}\right)$

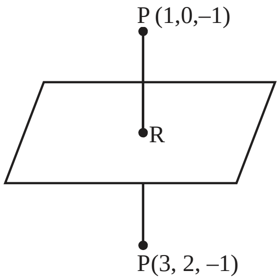
$\vec{r}=\hat{i}+\hat{k}+t\left(\hat{i}+\hat{j}-2\hat{k}\right)$

$\Rightarrow\frac{\ell}{1}=\frac{m}{1}=\frac{-2}{-2}\quad\Rightarrow\ell=m=1$

From (1) $\frac{1-\alpha}{1}=-1\quad\Rightarrow\alpha=2$

&$\frac{1-\gamma}{-2}=-1\quad\Rightarrow\ \gamma=-1$

Q.18 (A, B, C)



R is mid point of PQ

$\therefore$ R (2, 1, –1) and it lies on plane

equation of plane is $\alpha a+\beta y+\gamma z=\delta$

$\therefore\ 2\alpha+\beta-\gamma=\delta$ .....(1)

Normal vector to plane is

$\vec{n} = 2i + 2j$

$\therefore \frac{\alpha}{2} = \frac{\beta}{2} = \frac{\gamma}{0} = k$

$\therefore \alpha=2k, \beta=2k, \gamma=0$ .....(2)

and $\alpha+\gamma=1$ (given) .....(3)

from (2) and (3)

$\therefore \alpha=1, \beta=1, \gamma=0$

and from (1)

$2(1)+1-0=\delta$

$\delta=3$

Now :

$\alpha+\beta=2$

$\delta-\gamma=3$

$\delta+\beta=4$

so, A,B,C are correct.

Q.19 [1.00]

Q.20 [1.50]

19 & 20

$7x + 8y + 9z - (\gamma - 1) = A(4x + 5y + 6z - \beta) +$

$B(x+ 2y + 3z - \alpha)$

$x : 7 = 4A + B$

$y : 8 = 5A + 2B$

$A = 2, B = -1$

const. term : $-(\gamma - 1) = -A\beta - \alpha B \Rightarrow -(\gamma - 1)$

$\equiv 2\beta + \alpha$

$\alpha - 2\beta + \gamma = 1$

$$M = \begin{pmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \alpha - 2\beta + \gamma = 1$$

Plane P : $x - 2y + z = 1$

Perpendicular distance $= \left|\frac{3}{\sqrt{6}}\right| = P \Rightarrow D = P^2 = \frac{9}{6} = 1.5$

3-Dimensional Geometry

EXERCISES

ELEMENTARY

Q.1 (3)

$$2A+2B=\begin{bmatrix}2 & 0\\ 2 & 2\end{bmatrix},\ A-2B=\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}$$

On adding, we get $3A=\begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix}$

$$\Rightarrow A=\begin{bmatrix}1/3 & 1/3\\ 2/3 & 1/3\end{bmatrix}$$

Q.2 (3)

Clearly, $AB=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{bmatrix}$

$$=\begin{bmatrix}\cos(\alpha+\beta) & -\sin(\alpha+\beta)\\ \sin(\alpha+\beta) & \cos(\alpha+\beta)\end{bmatrix}=BA \text{ (verify).}$$

Q.3 (2)

$$(A-2I)(A-3I)=\begin{bmatrix}2 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=O$$

Q.4 (4)

$$A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$$

$$A^2=A.A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 3 & 0\\ -3 & 2 & -2\\ 6 & 4 & 5\end{bmatrix}$$

$$A.A^2=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}\begin{bmatrix}4 & 3 & 0\\ -3 & 2 & -2\\ 6 & 4 & 5\end{bmatrix}=\begin{bmatrix}4 & 11 & 1\\ -9 & -2 & -7\\ 21 & 11 & 7\end{bmatrix}$$

$$\Rightarrow A^3-3A^2-A+9I_3=0$$

Q.5 (4)

Given, Matrix $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

We know that

$$A^2=A.A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}.$$

Therefore

$$A^{16}=(A^2)^8=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}^8=\begin{bmatrix}(-1)^8 & 0\\ 0 & (-1)^8\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Q.6 (3)

Given $AB=A$, $\therefore$ $B=I$ $\Rightarrow BA=B$, $\therefore$ $A=I$.
Hence, $A^2=A$ and $B^2=B$.

Q.7 (1)

$$\begin{bmatrix}7 & 1 & 2\\ 9 & 2 & 1\end{bmatrix}\begin{bmatrix}3\\ 4\\ 5\end{bmatrix}=\begin{bmatrix}35\\ 40\end{bmatrix};$$

$$\therefore \begin{bmatrix}35\\ 40\end{bmatrix}+\begin{bmatrix}8\\ 4\end{bmatrix}=\begin{bmatrix}43\\ 44\end{bmatrix}$$

Q.8 (2)

$$A'=\begin{bmatrix}0 & -1 & 2\\ 1 & 0 & -5\\ -2 & 5 & 0\end{bmatrix}=-A$$

Q.9 (4)

Matrix $\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & \lambda & 5\end{bmatrix}$ be non singular,

only if $\begin{vmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & \lambda & 5\end{vmatrix}\neq 0$

$\Rightarrow$ $1(25-6\lambda)-2(20-18)+3(4\lambda-15)\neq 0$

$\Rightarrow$ $25-6\lambda-4+12\lambda-45\neq 0$

$\Rightarrow 6\lambda-24\neq 0$ $\Rightarrow$ $\lambda\neq 4$

Q.10 (2)

$$AB=\begin{bmatrix}1 & -2 & 1\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 2\\ 1 & 1\end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix} \Rightarrow (AB)^T = \begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$$

Q.11 (3)

$= \ (A - A^T)^T = A^T - (A^T)^T$

$= \ A^T - A \ [\because (A^T)^T = A] \ = -(A - A^T)$

So, $A - A^T$ is a skew symmetric matrix

Q.12 (2)

In A^{-1} the element of 2nd row and 3rd column is the c_{32} element of the matrix (c_{ij}) of cofactors of element of A, (due to transposition) divided by $\Delta = |A| = -2$.

$\therefore$ Required element = $\dfrac{(-1)^{3+2} M_{32}}{-2} \ = \dfrac{-(-2)}{-2} = -1$,

where $M_{32} =$ minor of c_{32} in A= $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = 0 - 2 = -2$

Q.13 (1)

Since A is symmetric, therefore $A^T = A$.

Now $(A^n)^T = (A^T)^n = (A)^n$

$\therefore$ A^n is also a symmetric matrix.

Q.14 (1)

In a skew-symmetrix matrix $a_{ij} = -a_{ji} \ \forall \ i, j = 1, 2, 3$

for $j = i$, $a_{ii} = -a_{ji} \ \Rightarrow \ a_{ii} = 0$

$\Rightarrow$ each .

Hence the matrix $\begin{bmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix}$ is skew-symmetric.

Q.15 (4)

Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |A| = 1$

$$\text{adj}(A) = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 1 \\ 7 & -2 & 1 \end{bmatrix}^T .$$

Hence, $A^{-1} = \dfrac{\text{adj}(A)}{|A|} \ \Rightarrow A^{-1} = \begin{bmatrix} 1 & -2 & 7 \\ 2 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$.

Hence, element $z = 3$.

Q.16 (1)

$3A^3 + 2A^2 + 5A + I = 0 \Rightarrow I = -3A^3 - 2A^2 - 5A$

$\Rightarrow \ IA^{-1} = -3A^2 - 2A - 5I$

$\Rightarrow \ A^{-1} = -(3A^2 + 2A + 5I)$

Q.17 (1)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |A| = -1(1+0) = -1$$

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$\Rightarrow \ A_{11} = 0, A_{12} = -1, A_{13} = 0$

$A_{21} = -1, A_{22} = 0, A_{23} = 0$

$A_{31} = 0, \ A_{32} = 0, A_{33} = -1$

$$\Rightarrow \ A^{-1} = \frac{\text{adj}(A)}{|A|} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$$

Q.18 (1)

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{|A|}.\text{adj}(A)$$

$\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = -1$; $|A| = 0 - 1(1-9) + 2(1-6) = 8 - 10$

$|A| = -2 \neq 0$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$A_{11} = (-1)^{1+1}[(2)(1) - (3)(1)] = -1$

$A_{12} = 8, A_{13} = -5, A_{21} = 1, A_{22} = -6$

$A_{23} = 3, A_{31} = -1, A_{32} = 2, A_{33} = -1,$

$$\therefore \ A^{-1} = \frac{1}{-2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

Q.19 (2)

We have, $A(\text{adj} A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

or $A(\text{adj} A) = 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 10I$ …..(i)

and $A^{-1} = \dfrac{1}{|A|}(\text{adj} A)$

$$A(\text{adj}\,A) = |A|I \qquad .....(\text{ii})$$

From equation (i) and (ii), we get $|A| = 10$.

Q.20 (3)

$$X = \begin{bmatrix} -x & -y \\ z & t \end{bmatrix};\ \text{adj } X = \begin{bmatrix} t & y \\ -z & -x \end{bmatrix}$$

$$\therefore \text{ Transpose of (adj (X))} = \begin{bmatrix} t & -z \\ y & -x \end{bmatrix}$$

Q.21 (4)

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix} = 1[3] + 1[6] + 1[-4] = 5$$

$$B = \text{adj } A = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$$

$$\text{adj } B = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix} = 5A \text{ and } C = 5A$$

$$C = \text{adj } B;\ |C| = |\text{adj } B|;\ \frac{|\text{adj}\,B|}{|C|} = 1.$$

Q.22 (3)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$$

Let c_{ij} be co-factor of a_{ij} in A.
Then co-factor of elements of A are given by

$$C_{11} = \begin{vmatrix} 4 & 9 \\ 8 & 27 \end{vmatrix} = 36, C_{21} = \begin{vmatrix} 2 & 3 \\ 8 & 27 \end{vmatrix} = -30,$$

$$C_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6$$

$$C_{12} = \begin{vmatrix} 1 & 9 \\ 1 & 27 \end{vmatrix} = -18, C_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 27 \end{vmatrix} = 24, C_{32} = \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -6$$

$$C_{13} = \begin{vmatrix} 1 & 4 \\ 1 & 8 \end{vmatrix} = 4, C_{23} = \begin{vmatrix} 1 & 2 \\ 1 & 8 \end{vmatrix} = -6, C_{33} = \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 2$$

$$\Rightarrow |A| = \begin{vmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{vmatrix}$$

$$\Rightarrow |\text{Adj}(A)| = 36(48-36) + 30(-36+24) + 6(108-96)$$

$$\Rightarrow |\text{Adj}(A)| = 144$$

Q.23 (4)

$$\text{We know } A\,\text{adj}(A) = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$$\therefore |A|.|\text{adj}(A)| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$\therefore |A|.\text{adj}|A| = |A|^3$
Now question gives $|A| = 8$
$\therefore 8.\text{adj}|A| = 8^3$ or $\text{adj}|A| = 8^2 = (2^3)^2 = 2^6$

Q.24 (1)

Since $A^2 = O$ (Zero matrix) and 2 is the least +ve integer for which $A^2 = O$. Thus, A is nilpotent of index 2.

Q.25 (1)

$$\text{Since for given } A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$$

$\Rightarrow AA^T = A^TA = I_{(3\times3)}$. Thus A is orthogonal.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (3)
It is a 12 elements matrices. Possible orders are 1 × 12, 12 × 1, 2 × 6, 6 × 2, 3 × 4 and 4 × 3.
∴ Number of possible orders is 6.

Q.2 (1)

$$\begin{bmatrix} x^2+x & x \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -x+1 & x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^2+x & x-1 \\ -x+4 & x+2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^2+x & x-1 \\ -x+4 & x+2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$$

On comparing
$x^2 + x = 0 \Rightarrow x = 0, -1$ $\qquad$ $x - 1 = -2 \Rightarrow x = -1$
$-x + 4 = 5 \Rightarrow x = -1$ $\qquad$ $x + 2 = 1 \Rightarrow x = -1$
Hence the value of x is – 1.

Q.3 (4)
Given, $A + A^T = I$

$$\text{So, } \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore \quad 2\cos\alpha = 1 \Rightarrow \cos\alpha = \dfrac{1}{2}$

$\therefore \quad \alpha = \dfrac{\pi}{3}$ or $\dfrac{5\pi}{3}$

So, number of values of $\alpha \in (0, \pi)$ are two.

Q.4 (4)

Let matrix $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ can commute with $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

So, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow$

$$\begin{bmatrix} a+b & a \\ c+d & c \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ a & b \end{bmatrix}$$

$\therefore$ On comparing, we get $b = c$, $a = b + d$

So matrix $= \begin{bmatrix} a & b \\ b & a-b \end{bmatrix}$.

Q.5 (4)

Matrix A has order (3×1) and Matrix B has order (3×3).

So multiplication AB is not possible.

Q.6 (1)

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$I\cos\theta + J\sin\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} +$

$$\begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Q.7 (1)

$$AB = \begin{bmatrix} 3ax^2 & 3bx^2 & 3cx^2 \\ a & b & c \\ 6ax & 6bx & 6cx \end{bmatrix}$$

Now, tr (AB) = tr (C)

$\Rightarrow \quad 3ax^2 + b + 6cx = (x + 2)^2 + 2x + 5x^2$

$3ax^2 + 6c + b = 6x^2 + 6x + 4$

$\Rightarrow \quad a = 2, b = 4, c = 1$

$\Rightarrow \quad a + b + c = 7.$

Q.8 (2)

A = diag (2, –1, 3), B = diag (–1, 3, 2) then A^2 B = ?

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix} ; B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix};$$

$$A^2B = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 18 \end{bmatrix} = \text{diag }(-4, 3, 18)$$

Q.9 (1)

$$A^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2^2 & 0 \\ 0 & 2^2 \end{bmatrix}, A^6 = \begin{bmatrix} 2^3 & 0 \\ 0 & 2^3 \end{bmatrix},$$

$$A^8 = \begin{bmatrix} 2^4 & 0 \\ 0 & 2^4 \end{bmatrix}$$

$\therefore \quad (A^8 + A^6 + A^4 + A^2 + I)\, V = \begin{bmatrix} 31 \\ 62 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 31 & 0 \\ 0 & 31 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 31 \\ 62 \end{bmatrix}$

$\Rightarrow \quad x = 1, y = 2$

$\therefore \quad xy = 2.$

Q.10 (4)

If A is n^{th} root of I_2, then $A^n = I_2$. Now,

$$A^2 = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix};$$

$$A^3 = A^2A\begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a^3 & 3ab \\ 0 & a^3 \end{bmatrix}$$

Thus, $A^n = \begin{bmatrix} a^n & nab \\ 0 & a^n \end{bmatrix}$

Now $A^n = I$

$$\Rightarrow \begin{bmatrix} a^n & nab \\ 0 & a^n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow a^n = 1, b = 0$

Q.11 (3)

Given $A^2 = A$.

Now

$(I + A)^3 - 7A$

$= I^3 + 3I^2A + 3IA^2 + A^3 - 7A = I + 3A + 3A + A - 7A$

$= I + O = I$

Q.12 (2)

We have,

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$

Q.13 (4)

Assume $\mathbf{C} = \mathbf{AB} - \mathbf{BA}$

If $\mathbf{C} = \mathbf{I}$

then trace $(\mathbf{C}) = 1 + 1 + \ldots\ldots.. + 1 = n$

But trace $(\mathbf{C}) = 0$

$(\because$ trace $(\mathbf{AB})$ = trace $(.\mathbf{BA})$)

Which is a contradiction

Hence no such ordered pair is possible.

Q.14 (1)

$A^{N+1} = (B + C)^{N+1}$

We can expand $(B + C)^{N+1}$ like binomial expansion as $BC = CB$.

$\therefore \quad (B + C)^{N+1} = {}^{N+1}C_0 B^{N+1} + {}^{N+1}C_1 B^N C + {}^{N+1}C_2 B^{N-1} C^2 + \ldots\ldots\ldots.. + C^{N+1}$.

$= B^{N+1} + (N + 1) B^N C + 0 + 0 \ldots\ldots.. + 0 = B^N (B + (N+1) C)$.

Q.15 (1)

$A^2 - 2A + I = 0$

$\Rightarrow \quad (A - I)^2 = 0$

$A^n = (A - I + I)^n = {}^nC_0 (A - I)^n + \ldots\ldots. + {}^nC_{n-2} (A - I)^2 \cdot I^{n-2} + {}^nC_{n-1}(A - I) \cdot I^{n-1} + {}^nC_n I^n$

$= 0 + 0 + \ldots\ldots\ldots. + 0 + n(A - I) + I = nA - (n - 1)I$

Q.16 (3)

$AB = B$

Premultiply both sides by B

$BAB = B^2 \qquad \Rightarrow AB = B^2$

$\Rightarrow B = B^2$

Similarly

$BA = A \qquad \Rightarrow ABA = A^2$

$\Rightarrow BA = A^2 \qquad \Rightarrow A = A^2$

Q.17 (2)

$AB^n = A\,B\,B\,B\,B\,B \ldots\ldots B$

$= (AB)\,B\,B\,B \ldots\ldots. B$

$= BB\,(AB)\,B\,B \ldots\ldots B$

$\vdots$

$= B^n A$

$(AB)^n = (AB)\,(AB)\,(AB) \ldots\ldots (AB)$

$= A(BA)\,(BA)\,(BA) \ldots\ldots (BA)\,B$

$= A(AB)\,(AB)\,(AB) \ldots\ldots (AB)B$

$= A^2 (BA)\,(BA)\,(BA) \ldots\ldots (BA)B^2$

$= A^2 (AB)\,(AB)\,(AB) \ldots\ldots (AB)\,B^2$

$= A^3 (BA)\,(BA)\,(BA) \ldots\ldots (BA)\,B^3$

$\vdots$

$= A^nB^n$

Q.18 (3)

For upper triangle matrix, elements below diagonal are zero

$\therefore \quad a_{ij} = 0$, where $i > j$

Q.19 (3)

Trace of $A = a_{11} + a_{22} + a_{33}$

For skew symmetric matrix $a_{11} = a_{22} = a_{33} = 0$

Trace of A = 0

Q.20 (1)

$p^2 - q^2 = r$;

$p = 3$

$q = 2, r = 5$

Q.21 (1)

$A^T = -A$

$\Rightarrow \quad (A^n)^T = (AAA \text{-------} A)^T = (A^T A^T A^T \text{-------} A^T)$

$= (A^T)^n$ for all $n \in N$

$(-A)^n = (-1)^n A^n$

$$\Rightarrow (A^n)^T = \begin{cases} A^n \text{ if } n \text{ is even} \\ -A^n \text{ if } n \text{ is odd} \end{cases}$$

Q.22 $P + P^T = 0 \Rightarrow$ P is skew symmetric matrix of order 2.

$$\text{Let} \quad P = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$$

$$2A = 4I - P = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$$

$$\Rightarrow \quad 2A = \begin{bmatrix} 4 & -a \\ a & 4 \end{bmatrix}$$

$|2A| = 16 + a^2$

$\Rightarrow \quad 4\,|A| = 16 + a^2 \Rightarrow |A| = 4 + \dfrac{a^2}{4}$

$\therefore \quad |A| \geq 4$ **Ans.**]

Q.23 (2)

$| 3 AB| = |A| . |3B|_{3\times 3} = (-1) . 3^3 |B| = -81$

Q.24 (4)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$|A| = 4$

$$B = \begin{bmatrix} a_{12}+a_{13} & a_{11}+a_{13} & a_{11}+a_{12} \\ a_{22}+a_{23} & a_{21}+a_{23} & a_{21}+a_{22} \\ a_{32}+a_{33} & a_{31}+a_{33} & a_{31}+a_{32} \end{bmatrix}$$

$$|B| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$= 4 \times 2 = 8$

Q.25 (1)

Let $a = \cos^{-1}x,\ b = \cos^{-1}y,\ c = \cos^{-1}z,$

$|A| = 0$

$$\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a + b + c)$$

$$\left[\frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}\right] = 0$$

$\Rightarrow\ a + b + c = 0 \qquad \Rightarrow\ x = y = z = 1$

Q.26 (3)

$$A = \begin{bmatrix} 2 & 0 \\ -a & 2 \end{bmatrix} \Rightarrow \text{adj } A = \begin{bmatrix} 2 & 0 \\ a & 2 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ a & 2 \end{bmatrix}$$

$$\therefore\ A^{-2} = A^{-1} \cdot A^{-1} = \frac{1}{16} \cdot \begin{bmatrix} 2 & 0 \\ a & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ a & 2 \end{bmatrix}$$

$$= \frac{1}{16}\begin{bmatrix} 4 & 0 \\ 4a & 4 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 \\ a/4 & 1/4 \end{bmatrix}$$

$\Rightarrow\ x = \dfrac{a}{4} \qquad \therefore\ \dfrac{a}{x} = 4.$ **Ans.**]

Q.27 (4)

$| \text{adj } A | = | A |^2 = 9$, and $\left|\text{adj adj}(2A)\right| = \left|(2A)\right|^4$

$= \left(2^3 | A |\right)^4$

$2^{12} \cdot |A|^4 = 2^{12} \cdot 9^2 = 24^2$

Q.28 (1)

From given data $|A| = 2^4$

$\Rightarrow\ |\text{adj (adj) } A| = (2^4) = 2^{36}$

$$\Rightarrow \left\{\frac{dt\left(adj(adj)A\right)}{7}\right\} = \left\{\frac{2^{36}}{7}\right\} = \left\{\frac{(7+1)^{12}}{7}\right\} = \frac{1}{7}$$

Q.29 (2)

A is a skew symmetric matrix $\therefore A^T = -A$

Now, $A^2 = -I$

$AAA^T = -A^T$

$AAA^T = A$

$A^{-1}AAA^T = A^{-1}A$

$AA^T = I \Rightarrow A$ is orthogonal matrix.

Q.30 (3)

$A^2 = A,\ I^2 = I$

$A = A^2 = A^3 = \ldots\ldots = A^n$

Same $I = I^2 = I^3 = \ldots\ldots = I^n$

$(I + A)^n = I^n + {}^nC_1\, I^{n-1} \cdot A + {}^nC_2\, I^{n-2} \cdot A^2 + \ldots\ldots + {}^nC_nA^n$

$= I + {}^nC_1 A + {}^nC_2 A + \ldots\ldots + {}^nC_nA$

$= I + ({}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n)A$

$= I + (2^n - 1) A$

$(I + A)^n = I + (2^n - 1)A = I + (2^n + k)A$

$\therefore\ K = -1$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (C)

$$A^2 = \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix}\begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix} A^2 =$$

$$\begin{bmatrix} a^2-1 & a+b \\ -(a+b) & b^2-1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence $a^2 - 1 = 0$; $\quad a + b = 0$

$b^2 - 1 = 0$

$a = 1$ or -1 ; $b = 1$ or -1

if $a = 1$ or $b = -1$

or $a = -1,\ b = 1$

These all the conditions are fulfilled

$\Rightarrow\ ab = -1$ Ans.]

Q.2 (A)

AB = 0

$$\Rightarrow \begin{bmatrix} \cos^2\theta\cos^2\phi + \sin\theta\sin\phi\cos\theta\cos\phi & \cos^2\theta\cos\phi\sin\phi + \sin^2\phi\cos\theta\sin\theta \\ \cos^2\phi\cos\theta\sin\theta + \sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi(\cos\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix} = 0$$

$$\Rightarrow \cos(\theta-\phi)\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \cos\phi\sin\theta & \sin\theta\sin\phi \end{bmatrix} = 0$$

$$\Rightarrow \cos(\theta-\phi) = 0 \Rightarrow \theta - \phi = (2n+1)\ \frac{\pi}{2}$$

Q.3 (A)

$R = P^TQ^8\,P = A^8$

Now, $A^2 = AA = \begin{bmatrix} \sqrt{3} & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{3} & -2 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & -2\sqrt{3}-2 \\ 0 & 1 \end{bmatrix}$

Also, $A^3 = A^2\ A = \begin{bmatrix} 3 & -2\sqrt{3}-2 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} \sqrt{3} & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \left(\sqrt{3}\right)^3 & -6-2\sqrt{3}-2 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad R = [r_{ij}]_{2\times 2} = P^T Q^8 P = A^8$

$= \begin{bmatrix} \left(\sqrt{3}\right)^8 & - \\ - & - \end{bmatrix} \Rightarrow \quad r_{11} = 81$

Q.4 (D)

$M(0) = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix};\ \left(M(0)\right)^T$

$= \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$\left(M(0)\right)\left(M(0)\right)^T$

$= \begin{bmatrix} a^2+b^2 & bc & -ac \\ bc & a^2+c^2 & ab \\ -ac & ab & b^2+c \end{bmatrix},$

which is symmetric matrix .

Q.5 (A)

$|A| \neq 0$ and $|B| \neq 0 \quad \Rightarrow |AB| \neq 0$

$\therefore$ AB is non singular.

Q.6 (B)

AB = AC

Pre-multiplying $A^{-1} \Rightarrow B = C$ hence A must be invertible matrix.

Q.7 (C)

We have $A(A + I) = -2I$

$\Rightarrow \quad |A(A + I)| = |-2I|$

$\Rightarrow \quad |A|\,|A + I| = 4 \neq 0$

Thus , $|A| \neq 0 \Rightarrow$ A is non singular

$\Rightarrow$ A is correct

Also , $A\left(-\frac{1}{2}(A+I)\right) = I$

$\Rightarrow \quad A^{-1} = -\frac{1}{2}(A+I)$

$\Rightarrow$ D is correct

Also A = 0 does not satisfy the given eqaution

$\Rightarrow \quad A \neq 0$

again $\left.\begin{matrix} A^2 + A + 2I = 0 \\ (A^T)^2 + A^T + 2I = 0 \end{matrix}\right]$ subtract again

will $A^T = B$

$(A^2 - B^2) + (A - B) = 0$

$(A - B)(A + B + I) = 0$

$\not\Rightarrow \quad A - B = 0 \quad$ or $\quad A + B + I = 0$

Q.8 (A)

$A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix formed by Cofactors of A= C =

$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad \text{Adj } A = C^T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = A'$

Q.9 (C)

$kA \text{ adj }(kA) = |kA|\ I_n$

$kA \text{ adj }(kA) = k^n\ |A|\ I_n$

$kA \text{ adj }(kA) = k^n\ A \text{ adj } A$

Pre-multiplying A^{-1}

$\text{adj }(kA) = k^{n-1} \text{ adj } A$

Q.10 (A)

$A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix};$

$A = BX \Rightarrow B^{-1}A = B^{-1}(BX) \Rightarrow B^{-1}A = X$

$X = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix};$

$X = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$

Q.11 (D)

$|F| \neq 0$ and $|G| \neq 0$

As we known $[AB]^{-1} = B^{-1} A^{-1}$

$\therefore\ [F(\alpha)\ G(\beta)]^{-1} = [G(\beta)]^{-1}[F(\alpha)]^{-1}$

Q.12 (C)

$|A| \cdot (\text{adj}\,(A^{-1})) = |A|\,(|A^{-1}| \cdot (A^{-1})^{-1}) = |A\,A^{-1}| \cdot A = A.$

Q.13 (C)

$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$; $\quad |A| = abc$

$\text{adj}\,(A) = \begin{bmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{bmatrix}$;

$A^{-1} = \dfrac{\text{adj}(A)}{(A)} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$

Q.14 (A)

$\because A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$ and A^{-1}

$= \dfrac{1}{\sec^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

$\therefore A^TA^{-1} = \dfrac{1}{\sec^2 x}\begin{bmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-\tan^2 x \end{bmatrix}$

$|A^TA^{-1}| = \dfrac{1}{\sec^4 x}\,(1 + \tan^2 x)^2 = 1.$

Q.15 (B)

$|A| \neq 0$

$\Rightarrow\ a(ed - 0) + b\,(0 - ce)$

$|A| = aed - bce = e\,(ad - bc) \neq 0$

$e = 1$ and $ad - bc \neq 0$

$ad - bc = 1$

if $[ad = 1, bc = 0]$ Total = 3

$ad - bc = -1$

if $[ad = 0, bc = 1]$ Total = 3

Total = 3 + 3 = 6

Q.16 (A)

$A^2 - 2A + I = 0 \Rightarrow (A - I)^2 = 0$

$A^n = (A - I + I)^n = {}^nC_0\,(A - I)^n + \ldots\ldots. + {}^nC_{n-2}\,(A - I)^2 \cdot I^{n-2} + {}^nC_{n-1}(A - I) \cdot I^{n-1} + {}^nC_n\,I^n$

$= 0 + 0 + \ldots\ldots\ldots. + 0 + n(A - I) + I = nA - (n - 1)I$

Q.17 (A)

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{bmatrix} = 0$$

$\Rightarrow (1 - \lambda)\,[(2 - \lambda)\,(3 - \lambda)] + 2[-2(2 - \lambda)] = 0$

$\Rightarrow (1 - \lambda)\,[\lambda^2 - 5\lambda + 6] + 4\,(\lambda - 2) = 0$

$\Rightarrow\ \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0$

$\Rightarrow\ x^3 - 6x^2 + 7x + 2 = 0$

$\Rightarrow\ k = 2$

Q.18 (C)

$\because\ A^2 = A$ and $|A| \neq 0$

$\therefore A^{-1} A.A = A^{-1} A$

$\Rightarrow A = I$

$\therefore |A| = 1$ and tr (A) = 3

$\therefore$ Given sum $= 1 + \dfrac{1}{3} + \dfrac{1}{3^2} + \ldots\ldots$

$= \dfrac{1}{1-\frac{1}{3}} = \dfrac{3}{2}.$

JEE-ADVANCED MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (ABD**)**

$|A| = 18$

$\text{adj}\,(\text{adj}\,A) = |A|^{n-2} A = 18A = \begin{bmatrix} 54 & 0 & 0 \\ 36 & 36 & 0 \\ 72 & 90 & 54 \end{bmatrix}$

$\Rightarrow \text{trace}\big(\text{adj}\,(\text{adj}\,A)\big) = 144.$

$|\text{adj}\,A| = |A|^{n-1} = 18^2 = 324$

Q.2 (CD**)**

$|A| \neq 0$

$|B| \neq 0$

$9A^2B - 6AB = \begin{bmatrix} 9 & 27 \\ -9 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}$;

$(9A^2 - 6A + I\,) = \begin{bmatrix} 6 & 27 \\ -10 & -1 \end{bmatrix} B^{-1}$

$9A^2 - 6A + I = \begin{bmatrix} 6 & 27 \\ -10 & -1 \end{bmatrix} \dfrac{\begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}}{3}$;

$$(3A - I)^2 = \begin{bmatrix} -6 & 27 \\ -3 & 0 \end{bmatrix}$$

$|3A - I|^2 = 81$

$|3A - I| = \underline{+}\, 9$

Q.3 (ABC)

We have $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = B$ (say)

Now, $A^{-n} = B^n = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

$\Rightarrow \frac{1}{n} A^{-n} = \begin{bmatrix} 1/n & 0 \\ -1 & 1/n \end{bmatrix}$ and $\frac{1}{n^2} A^{-n}$

$= \begin{bmatrix} 1/n^2 & 0 \\ -1/n & 1/n^2 \end{bmatrix}$

$\Rightarrow \underset{n\to\infty}{\text{Limit}} \frac{1}{n} A^{-n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$ and $\underset{n\to\infty}{\text{Limit}} \frac{1}{n^2} A =$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Q.4 (BC)

We have $|A^{-1}| = \begin{vmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{vmatrix} = 2$, therefore, $|A| = 1/$

2

Since $A^{-1} = \frac{1}{|A|}$ (Adj. A) we get

Adj. $A = |A|A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{bmatrix}$

A cannot be skew symmetric as |A| = 0 for all skew symmteric matrices of order (2n + 1) x (2n + 1)

Q.5 (AD)

The elements of main diagonal of skew symmetric matrix are all zero but not necessarily for symmetric matrix.

Q.6 (CD)

(A) Skew-symmetric matrix of even order can be invertible also.

e.g. $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

(B) If AB = **O** $\not\Rightarrow$ that one of the matrices is zero.

e.g. $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 5 \\ 0 & 0 \end{bmatrix} \Rightarrow AB =$ **O**.

(C) Minimum number of cyphers in an upper triangular matrix of order n is $\frac{n(n-1)}{2} = 5050 \Rightarrow$ n = 101.

(D) We have $|\,10\,AB\,| = 10^3\,|A|\,|B| = (10^3)(5)(2) = 10^4$.

Q.7 (AC)

$(A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $bc \neq 0$

Characteristic equation is $|A - xI| = 0$

$\begin{vmatrix} a-x & b \\ c & d-x \end{vmatrix} = 0$ $(a - x)(d - x) - bc = 0$

$x^2 - x(a + d) + ad - bc = 0$

On comparing with the given equation $x^2 + k = 0$

$a + d = 0$, $k = ad - bc = |A|$

Q.8 (B C)

$(A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$A^2 - 4A + 5I_2$

$\Rightarrow \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

= 0

$C = A - B = \begin{bmatrix} 1-\alpha & 0 \\ 0 & -2 \end{bmatrix}$ is diagonal matrix,

$\forall\ \alpha \in R$

Q.9 (AB)

(A) Given that AB = **O**, where det. (A) ≠ 0 ....(1)

So, A^{-1} exists.

Now, pre-mutiplying equation (1) with A^{-1}, we get $(A^{-1}A)\,B = A^{-1}\mathbf{O} \Rightarrow B = \mathbf{O}_{\text{null matrix}}$.

(B) Given, det. (A) = 2, det. (B) = 3, det. (C) = 4

So, det. (3ABC) = 3^2 det. (A) det. (B) det. (C) = 9(2) (3) (4) = 216. **Ans.**

(As, A, B, C are square matrices of order 2.)

(C) Given, det. (A) = $\frac{1}{2}$ (order of matrix A is 3)

As, det. (adj. A) = $(\text{det. } A)^{n-1}$ ......(1)

place A by A^{-1} in equation (1) and take n = 3, we get

det (adj. A^{-1}) = $\left|A^{-1}\right|^2 = \frac{1}{|A|^2} = \frac{1}{\left(\frac{1}{2}\right)^2} = 4$. **Ans.**

(D) We know that skew symmetric matrix of odd order is singular. But , if order of skew symmetric matrix is even, then it need not be singular. For example,

$$A = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} \text{ and det. } A = 16 \text{ (non - singular).}$$

Q.10 (ACD)

(A) $$\Delta = \begin{vmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{vmatrix}$$

A = 0, singular

And (adj A) B = 0 system is inconsistent

(B) Obviously the possible orders are 1 × 6, 2 × 3, 3 × 2 and 6 × 1.

No. of possible orders is 4

(C) $A\,(\text{adj } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \Rightarrow A\,(\text{adj } A) = 10$

$\begin{bmatrix} 1 & 0 \\ 1 & 10 \end{bmatrix} \Rightarrow A\,(\text{adj } A) = |A| I_n$

$\therefore\ |A| = 10$

(D) if C = B′AB

C′ = (B′ AB)′ = B′A (B′)′ = B′A′B = – B′AB

here A′ = – A

Q.11 (BD)

$M = ABCD = A(BCD) = AA^T$

M^3 = (ABCD) (ABCD) (ABCD) = (ABC) (DAB) (CDA) (BCD) = $D^T\,C^T\,B^T\,A^T = (BCD)^T A^T = AA^T$ = M.

Similarly M^{9k} = M.

Q.12 (AC)

Taking $C_3 \to C_3 - (C_1\alpha - C_2)$

we get

$$|A| = \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha+1 \end{vmatrix} = (1 - 2\alpha)\,(ac - b^2)$$

$\therefore$ non-invertible if $\alpha = \frac{1}{2}$ or if a, b, c are in G.P.

Q.13 (AC)

$\mathbf{M}^{-1}$ adj(adj $\mathbf{M}$) = $\mathbf{k}^2\mathbf{I}$

Pre-multiplying by **M**

adj(adj **M**) = $\mathbf{k}^2\mathbf{M}$

det(adj(adj M)) = det($\mathbf{k}^2\mathbf{M}$)

$(\det \mathbf{M})^{(3-1)^2} = \mathbf{k}^6 \det(\mathbf{M})$

$(\det \mathbf{M})^4 = \mathbf{k}^6 (\det \mathbf{M})$

$(\det \mathbf{M})^3 = \mathbf{k}^6$

$\det \mathbf{M} = \mathbf{k}^2 \Rightarrow k^2 = \alpha \qquad \Rightarrow k^2 = 4$

Q.14 (ABCD)

| A | = 6, | adj A | = $|A|^2$ = 36

$|\text{adj (adj } A)| = |A|^{(n-1)^2} = 6^4 = 1296$

adj (adj A) = $|A|^{(n-2)} \cdot A = |A| \cdot A$ {$\because$ n = 3}

$\therefore$ tr (adj (adj A)) = tr (6A) = 36

adj (adj A) = | A |· A

adj (adj (adj A)) = |adj A| adj A

A· adj (adj (adj A)) = |adj A| A adj A = |adj A| | A |· I_3

$\therefore$ tr (A adj (adj (adj A))) = $3 \cdot 6^2 \cdot 6 = 2^3 \cdot 3^4$

Q.15 (ABD)

$$A = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix}; \text{adj.}$$

$$A = \begin{pmatrix} d_2d_3 & 0 & 0 \\ 0 & d_1d_3 & 0 \\ 0 & 0 & d_1d_2 \end{pmatrix}$$

Let C = adj. A satisfy $x^3 - 9x + px - 27 = 0$

$\therefore$ Tr. (C) = 9 and

det. C = 27 $\Rightarrow$ | adj. A | = 27 $\Rightarrow |A|^2 = 27 \Rightarrow |A| = 3\sqrt{3}$

Also, |adj. (adj. A)| = $|A|^4 = 3^6$

Now, Tr.(adj. A) = 9 given

$\therefore$ $d_1d_2 + d_2d_3 + d_3d_1 = 9$ ……(1)

and $|A| = d_1d_2d_3 = 3\sqrt{3}$ ……(2)

$\Rightarrow \quad \frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} = \frac{9}{3\sqrt{3}} = \sqrt{3}$ = Tr.(A^{-1})

Also, G.M. of d_1; d_2; d_3 is $\sqrt{3}$

and H.M. of d_1, d_2, d_3 is $\sqrt{3}$

$\Rightarrow$ Tr.(A) = $3\sqrt{3}$

Comprehension # 1 (Q. No. 16 to 18)

Q.16 (AC)

Let any symmetric matrix B = $\begin{bmatrix} a & x & y \\ x & b & z \\ y & z & c \end{bmatrix}$

so total matrices possible = 3 ! × 3 ! = 36.

Q.17 (AC)

For homogeneous system infinite solution are possible if |B| = 0

$|B| = abc + 2xyz - by^2 - cx^2 - az^{2.}$

abc = xyz = 0 (one of a,b,c and one of x, y, z is zero)

if a = 0	z = 0
⇒ |B| = 0	- 4 cases
b = 0	y = 0
⇒ |B| = 0	- 4 cases
c = 0	x = 0
⇒ |B| = 0	- 4 cases

so total cases are 12.

Q.18 (AC)

BX = V is always inconsistent. when |B| = 0

so 12 cases

Comprehension # 2 (Q. No. 19 to 21)

Q.19 (A)

AB = A

Premultiplying by B

BAB = BA

BB = B

$B^2 = B$

⇒ B is idempotent

similarly on post mutliplying by A

$ABA = A^2$

$\Rightarrow AB = A^2$

$A = A^2$

⇒ A is idempotent

Q.20 (D)

For orthogonal matrix AA′ = I

$$\Rightarrow \begin{vmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{vmatrix} \begin{vmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2 \\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$\Rightarrow 4\beta^2 + \gamma^2 = 1,\ 2\beta^2 - \gamma^2 = 0,\ -2\beta^2 + \gamma^2 = 0,\ \alpha^2 - \beta^2 - \gamma^2 = 0,\ \alpha^2 + \beta^2 + \gamma^2 = 1$

$$\therefore \quad \alpha = \pm\frac{1}{\sqrt{2}},\ \beta = \pm\frac{1}{\sqrt{6}},\ \gamma = \pm\frac{1}{\sqrt{3}}$$

Q.21 (C)

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \qquad \Rightarrow A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow A^2$ is nilpotent

Comprehension # 3 (Q. No. 22 to 24)

Q.22 (A)

$$\underbrace{[3 \quad 4 \quad 5]}_{1\times3}\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}}_{3\times3} = \underbrace{[21 \quad 20 \quad 29]}_{\text{Pythagoren triplet}}$$

|||ly [3 4 5] [B] = [5 12 13]

and [3 4 5] [C] = [15 8 17]

hence question no.(i) answer is (D)

Q.23 (D)

det. A = 1 ; det. B = 1, det. C = 1

(verify)

det. (AB) = (det. A) (det. B) = (1) (1) = 1

det. (BC) = (det. B) (det. C) = (1) (1) = 1

det. (CA) = (det. C) (det. A) = (1) (1) = 1

det. (ABC) = (det. A) (det. B) (det. C) = 1

Q.24 (A)

$T_r(A + B^T + 3C) = \sum a_{ii} + \sum b_{ii} + 3\sum c_{ii} = 5 + 3 + 9 = 17$ **Ans. is (A)**

Comprehension # 5 (Q. No.25 to 27)

Q.25 (A)

$(A + B)C = (A + B)(A + B)^{-1}(A - B)$

$\Rightarrow \quad (A + B)C = A - B \qquad ....(1)$

$C^T = (A - B)^T\left((A+B)^{-1}\right)^T$

$= (A + B)\left((A+B)^T\right)^{-1} \qquad \{\text{as } |A + B| \neq 0$

$\Rightarrow \quad |(A + B)^T| \neq 0 \quad \Rightarrow \quad |A - B| \neq 0\}$

$= (A + B)(A - B)^{-1} \qquad ....(2)$

(1) & (2)

$C^T(A + B)C = (A + B)(A - B)^{-1}(A - B)$

$= (A + B) \qquad ....(3)$

Q.26 (B)

taking transpose in (3)

$C^T(A + B)^T(C^T)^T = (A + B)^T$

$C^T(A - B)C = A - B \qquad ....(4)$

Q.27 (C)

adding (3) and (4)

$C^T[A + B + A - B]C = 2A$

$C^TAC = A$

Q.28 $(A) \to (q)$; $(B) \to (p)$; $(C) \to (s)$; $(D) \to (q)$

(A) $|(A^{-1}) \text{ adj } (B^{-1}) \,.\, \text{adj}(2A^{-1})| = |A^{-1}| \,.\, |\text{adj } B^{-1}| \,.\, |\text{adj} \,.\, 2A^{-1}| = \frac{1}{|A|}.|B^{-1}|^2|2A^{-1}|^2 = \frac{2^6}{8} = 8$

(B) $I^3 + 3I^2A + 3IA^2 + A^3 = I + 3A + 3A + A = I + 7A$
$\Rightarrow k = 7$

(C) $AB = C \Rightarrow \det(A)\det(B) = \det(C) \Rightarrow \det(B) = -1$

(D) $A \text{ Adj } A = |A| I_3 = kI_3 \Rightarrow k = |A| = 8$

Q.29 $(A) \to p$; $(B) \to s$; $(C) \to p$; $(D) \to p$

$R = P^TQ^KP$

$= P^T(PAP^T)^K P$

$$= P^T \underbrace{PAP^TPAP^T..............PAP^T}_{\text{K times}} P$$

$= A^K$ as $PP^T = I$ as P is orthogonal

$$R = A^K = \begin{bmatrix} 1+2K & -4K \\ K & 1-2K \end{bmatrix}$$

|||ly $T = P^TS^KP = B^K$

$$B^K = \begin{bmatrix} a^K & \dfrac{b(a^K-1)}{a-1} \\ 0 & 1 \end{bmatrix}$$

NUMERICAL VALUE BASED

Q.1 [5049]

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

$$\begin{bmatrix} ap+bq \\ cp+dq \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

hence $ap + bq = p$ .....(1)

and $cp + dq = q$ .....(2)

or $p(a - 1) + bq = 0$

$cp + (d - 1)q = 0$

for non trivial solution of p and q

$$\begin{vmatrix} a-1 & b \\ c & d-1 \end{vmatrix} = 0$$

$ad - a - d + 1 - bc = 0$

$ad - bc = a + d - 1 = 5049$

Q.2 [2]

Let $A = \text{diag. }(a, b, c)$

$\therefore \quad A^{-1} = \text{diag.}\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$

$\Rightarrow 2A^2 - B = \lambda I \Rightarrow \quad 2A^2 - 4A^{-1} = \lambda I$

$\Rightarrow \quad \text{diag.}\left(2a^2 - \frac{4}{a}, 2b^2 - \frac{4}{b}, 2c^2 - \frac{4}{c}\right) = (\lambda, \lambda, \lambda)$

$\Rightarrow 2x^2 - \frac{4}{x} = \lambda \Rightarrow 2x^3 - \lambda x - 4 = 0 \left< \begin{matrix} a \\ b \\ c \end{matrix} \right.$

$\Rightarrow a + b + c = 0, abc = 2$

Tr. (A) = 0 and Det.(A) = 2

$\therefore \ p + q = 2$

Q.3 [0035]

$$AB = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix} =$$

$$\begin{bmatrix} 3p+2r & 3q+2s \\ -p+4r & -q+4s \end{bmatrix}$$

$AB = \text{diag }(d_{11}, d_{22})$

$$\begin{bmatrix} 3p+2r & 3q+2s \\ -p+4r & -q+4s \end{bmatrix} = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}$$

$3p + 2r = d_{11}$; $\quad 3q + 2s = 0 \Rightarrow 2s = -3q$

$-q + 4s = d_{22}$; $\quad -p + 4r = 0 \Rightarrow p = 4r$

$\therefore \ (d_{11} + d_{22}) = 12r + 2r + [-q + 2(-3q)]$

$= 14r - 7q = 7(2r - q)$

$\therefore |q + 1| + \sqrt{r-2} = 0 \Rightarrow q = -1, r = 2$

$\therefore \ (d_{11} + d_{22}) = 7(4 + 1) = 35.$

Q.4 [200]

$$\text{Consider}\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 2a+8 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 3a+24 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}^n$$

$$= \begin{bmatrix} 1 & 2n & na + 8\sum_{k=0}^{n-1} k \\ 0 & 1 & 4n \\ 0 & 0 & 1 \end{bmatrix}^n$$

hence n = 9 and

$$2007 = 9a + 8\sum_{k=0}^{8} k = 9a + 8\left(\frac{8 \cdot 9}{2}\right)$$

$2007 = 9a + 32 \cdot 9 = 9(a + 32)$

$a + 32 = 223 \quad \Rightarrow \quad a = 191$

hence $a + n = 200$

Q.5 [0005]

Sol. adj. A = – A $\quad \Rightarrow \quad$ A · adj. A = – A^2

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

So, $A^2 = I$

$$\Rightarrow \quad \begin{bmatrix} a^2 + bc & (a+d)b \\ (a+d)c & d^2 + bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

∴ On comparing, we get

$a^2 + bc = 1, \quad (a + d)b = 0, \quad (a + d)c = 0$,

$d^2 + bc = 1$

Case-I: When $(a + d) \neq 0$

⇒ b = 0 = c and a = 1, d = 1 or a = – 1, d = – 1

$$\therefore \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

but both rejected as det. A = – 1 (given.)

Case-II: When (a + d) = 0 ⇒ d = – a

(i) If a = 1, d = – 1 ⇒ bc = 0

For b = 0, c can be – 1, 0, 1.

For b = 1, c can be 0 only.

For b = – 1, c can be 0 only.

⇒ 5 matrices

(ii) If a = – 1, d = 1 ⇒ bc = 0

For b = 0, c = – 1, 0, 1.

For b = 1, c = 0 only.

For b = – 1, c = 0 only.

⇒ 5 matrices

(iii) If a = 0, d = 0 ⇒ bc = 1

$$\therefore \; A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ or } A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

⇒ 2 matrices

∴ N = 5 + 5 + 2 = 12

⇒ (N – 7) = 12 – 7 = 5

Q.6 **[0039]**

$$\left\{\frac{1}{2}\left(A - A' + I\right)\right\}^{-1} \text{ for } A = \begin{bmatrix} -2 & 3 & 4 \\ 5 & -4 & -3 \\ 7 & 2 & 9 \end{bmatrix}$$

$$\frac{1}{2}(A - A^T + I)^{-1} = \left[\frac{1}{2}\begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & -5 \\ 3 & 5 & 1 \end{bmatrix}\right]^{-1} = \left(\frac{1}{2}B\right)^{-1}$$

$$\Rightarrow \left|\frac{1}{2}B\right| = \frac{39}{8}$$

$$\text{Adj B} = \frac{1}{4}\begin{bmatrix} 26 & -17 & 7 \\ -13 & 10 & -11 \\ 13 & -1 & 5 \end{bmatrix}^T = \frac{1}{4}$$

$$\begin{bmatrix} 26 & -13 & 13 \\ -17 & 10 & -1 \\ 7 & -11 & 5 \end{bmatrix} \Rightarrow \left|\frac{1}{2}B^{-1}\right| =$$

$$\frac{2}{39}\begin{bmatrix} 26 & -13 & 13 \\ -17 & 10 & -1 \\ 7 & -11 & 5 \end{bmatrix}$$

Q.7 **[0017]**

$$A = \begin{vmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{vmatrix} \quad ; \quad |A| = \begin{vmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{vmatrix} \neq 0$$

$6\alpha - 5\alpha^2 \neq = 0 \quad \Rightarrow \quad \alpha(6 - 5\alpha) \neq 0$

$\alpha = 0, 6/5 \quad \therefore \quad \alpha \in R - \{0, 6/5\}$

For $\alpha = 1$

$$A = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} \Rightarrow |A| = 6 - 5 = 1 \; ;$$

$$\text{Adj A} = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{vmatrix}$$

$$\therefore A^{-1} = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & 5 \\ 5 & -2 & 2 \end{vmatrix}$$

By characteristic equation $|A - xI| = 0$

$$\begin{vmatrix} 2-x & 0 & -1 \\ 5 & 1-x & 0 \\ 0 & 1 & 3-x \end{vmatrix} = 0$$

$\Rightarrow \quad x^3 - 6x^2 + 11x - 1 = 0$

By cayley hamilton theorem

$A^3 - 6A^2 + 11A = I$

$\Rightarrow \quad A^{-1} = A^2 - 6A + 11.\ I$

Q.8 [0006]

$A^3 + 3A^2B + 3AB^2 + B^3 = (A + B)^3$

$\Rightarrow AB = BA$

$$AB = \begin{bmatrix} 4 & a \\ -1 & b \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4+a & -8+4a \\ -1+b & 2+4b \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 4 & a \\ -1 & b \end{bmatrix} = \begin{bmatrix} 6 & a-2b \\ 0 & a+4b \end{bmatrix}$$

$AB = BA$

$\Rightarrow 4 + a = 6 \Rightarrow a = 2$

$\Rightarrow -1 + b = 0 \Rightarrow b = 1$

$$AB = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = 6I$$

$\therefore\ B = 6A^{-1}$ and $A = 6B^{-1}$

$\Rightarrow A + B = 6A^{-1} + 6B^{-1} = 6\,(A^{-1} + B^{-1}) \equiv \lambda\,(A^{-1} + B^{-1})$

$\Rightarrow \lambda = 6.$

Q.9 [0009]

$B = \text{adj.}\,(2A) = 2\ \text{adj.}(A)$

$|B| = 4\,|\text{adj } A| = 4\,|A|$

$\therefore \quad |A| = 2$

$ad - bc = 2 \quad \Rightarrow \quad ad = 2 \quad \{\because bc = 0\}$

$$\text{adj.}\,(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\therefore\ B = \begin{bmatrix} 2d & -2b \\ -2c & 2a \end{bmatrix}$$

$$A + B = \begin{bmatrix} a+2d & -b \\ -c & 2a+d \end{bmatrix}$$

$\therefore\ \text{tr.}(A + B) = 3a + 3d = 3\,(a + d)$

$\therefore\ |\text{tr.}\,(A + B)| = 3 \times 3 = 9$

Ans. $\{\because\ a + d = 3$ or $-3\}$

Q.10 [650]

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix};\ B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

n(A) = number of elements in A

if (XY) is not defined then n(XY) = 0

C = (AB)(B'A)

$$AB = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ 11 \end{bmatrix}$$

$$B'A = \underbrace{\begin{bmatrix} 5 & -3 \end{bmatrix}}_{1\times 2} \underbrace{\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}}_{2\times 2} = \begin{bmatrix} -7 & 1 \end{bmatrix}$$

$$C = (AB)(B'A) = \begin{bmatrix} -1 \\ 11 \end{bmatrix} \begin{bmatrix} -7 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ -77 & 11 \end{bmatrix}$$

$\therefore \quad n(C) = 4; \quad n(A) = 4; \quad n(B) = 2$

$$D = (B'A)(AB) = \begin{bmatrix} -7 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 11 \end{bmatrix} = (7 + 11) = (18)$$

$\therefore \quad |\,D\,| = 18 \Rightarrow n(D) = 1$

$$\therefore \quad \left(\frac{n(C)\left(|\,D\,|^2 + n(D)\right)}{n(A) - n(B)} \right)$$

$$= \frac{4(324+1)}{4-2} = 650$$

KVPY

PREVIOUS YEAR'S

Q.1 (B)

$$A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix};\ A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix};\ A^3 = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix};\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I + A + A^2 + A^3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad ; A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$I + A + A^2 + A^3 + \ldots\ldots A^{2010}$$

$$\left(I + A + A^2 + A^3\right) + A^4\left(I + A + A^2 + A^3\right) + \ldots. + A^{2008}\left(I + A + A^2\right)$$

$$= \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

Q.2 (D)

$P^2 = P$

$P^{-1}P^2 = P^{-1}P$

$P = I$

$(I + P)^n = (2P)^n = 2^nP^n$

$= 2^nP$

$= P + (2^n - 1)\,P$

$= I + (2^n - 1)\,P$

Q.3 (A)

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a & b \\ c & d \end{bmatrix} ad = bc \Rightarrow \frac{a}{b} = \frac{c}{d} = k$$

$$\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} \quad a = bk\,c = dk$$

$a^2 + bc = a$; $b(a + d) = b$

$\Rightarrow a + d = 1$

Q.4 (C)

Subtracting the given equations we get $5x + 3y = 100$

$$x = 20 - \frac{3y}{5}$$

$\Rightarrow$ y is multiple of 5, let $y = 5k$

$x = 20 - 3k$

$\therefore$ $k = 0, 1, 2, ...6$

Hence numbers of solutions are 7.

Q.5 (C)

$|A| \neq 0 \Rightarrow a - b \neq 0$

$\Rightarrow a \neq b$ ...(i)

$$\text{Also, } A^{-1} = \frac{1}{a-b}\begin{bmatrix} 1 & -1 \\ b & a \end{bmatrix}^T$$

$$= \frac{1}{a-b}\begin{bmatrix} 1 & -b \\ -1 & a \end{bmatrix}$$

Thus, $a - b = 1$ or -1 ...(ii)

So, required number of pairs (a, b) is $101 \times 2 = 202$

Q.6 (B)

Sum of elements in each row of A is 1.

$$\text{So, } A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow A^{-1}A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = A^{-1}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = A^{-1}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Q.7 (B)

$$(2^n)_2 = 1\underbrace{00...0}_{n \text{ times}}$$

$M^2 = (2^{60} - 2^{46}) + (2^{30} - 2^{16}) + 2^{31} + 1$

$$\left(\underbrace{11...1}_{14 \text{ times}}\underbrace{00...0}_{46 \text{ times}} + \underbrace{11...1}_{14 \text{ times}}\underbrace{000...0}_{16 \text{ times}} + 1\underbrace{00...0}_{31 \text{ times}} + 1\right)_2$$

Number of 1's = 14 + 1 + 14 + 1 = 30

JEE-MAIN PREVIOUS YEAR'S

Q.1 (4)

$$A = \begin{bmatrix} 0 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 0 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I + A = \begin{bmatrix} 0 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}, I - A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix},$$

$$(I - A)' = \begin{bmatrix} 0 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$\therefore$ (I + A) (I – A)’ =

$$\begin{bmatrix} 0 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \tan^2\frac{\theta}{2} & 2\tan\frac{\theta}{2} \\ -2\tan\frac{\theta}{2} & 1 - \tan^2\frac{\theta}{2} \end{bmatrix}$$

$$\therefore a = 1 - \tan^2\frac{\theta}{2}, b = -2\tan\frac{\theta}{2}$$

$$\therefore 13(a^2 + b^2) = 13\left(\left(1 - \tan^2\frac{\theta}{2}\right)^2 + 4\tan^2\frac{\theta}{2}\right)$$

$$= 13\left(1 - \tan^2\frac{\theta}{2}\right)^2 = 13\sec^4\frac{\theta}{2}$$

Q.2 [21]

Q.3 [7]

$$A^2 = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow x^2 + y^2 + z^2 = 1$

$\Rightarrow x + y + z = 1$

$\Rightarrow xy + yz + zx = 0$

$|A|^2 = |I| \Rightarrow |A| = \pm 1 \Rightarrow 3xyz - (x^3 + y^3 + z^3) = \pm 1$

$x^3 + y^3 + z^3 = 3.2 \pm 1 = 7, 5$

$\Rightarrow x^3 + y^3 + z^3 = 7$

Q.4 (1)

$$D = \begin{vmatrix} 2 & +3 & 2 \\ 4 & 6 & 2 \\ -1 & 1 & 2 \end{vmatrix} = 2(10) - 3(10) + 2(10) \neq 0$$

so unique solution

Q.5 (1)

$|A| = 4$

$|2A| = 2^3\,|A| = 8 \times 4$

Now $R_2 \rightarrow 2R_2 + 5R_3$

$|B| = 2 \times 32 = 64$

Q.6 [1]

$AA^T = I$

$$\Rightarrow \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+\alpha^2 & \alpha - \alpha\beta \\ \alpha - \alpha\beta & \alpha^2 + \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \ 1 + \alpha^2 = 1 \Rightarrow \alpha = 0$

$\alpha^2 + \beta^2 = 1 \Rightarrow \beta^2 = 1$

$\therefore \ \alpha^4 + \beta^4 = 0 + 1 = 1$

Q.7 (1)

$A^T = A,\ B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2$

$\Rightarrow$ P is skew-symmetric matrix $\Rightarrow$ $|P| = 0$

Hence PX = 0 have infinite solution

Q.8 (1)

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a^2 + b^2 & b(a+c) \\ b(a+c) & b^2 + c^2 \end{bmatrix}$$

$tr(A^2) = a^2 + 2b^2 + c^2 = 1$

$\Rightarrow b = 0$ and $a^2 + c^2 = 1$

$\Rightarrow (a, c) = (1, 0),\ (-1, 0),\ (0, 1),\ (0, -1)$

Q.9 [540]

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

${}^9C_2 = 36$

Case- II : One (2) and three (1's) and five (0's)

$$\frac{9!}{5!3!} = 504$$

$\therefore$ Total = 540

Q.10 [17]

As $PQ = kI \quad \Rightarrow \quad Q = kP^{-1}$

now $Q = \dfrac{k}{|P|}(adjP)I \quad \Rightarrow \quad Q = \dfrac{k}{(20 + 12\alpha)}$

$$\begin{bmatrix} - & - & - \\ - & - & (-3\alpha - 4) \\ - & - & - \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\because q_{23} = \dfrac{-k}{8} \Rightarrow \dfrac{k}{(20 + 12\alpha)}(-3\alpha - 4)$

$= \dfrac{-k}{8} \Rightarrow 2(3\alpha + 4) = 5 + 3\alpha$

$3\alpha = -3 \Rightarrow \alpha = -1$

also $|Q| = \dfrac{k^3\,|I|}{|P|} \Rightarrow \dfrac{k^2}{2} = \dfrac{k^3}{(20 + 12\alpha)}$

$(20 + 12\alpha) = 2k \quad \Rightarrow 8 = 2k \Rightarrow k = 4k$

Q.11 [4]

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

•
•
•
•
•

$$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}, A^{20} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

L.H.S.

$$= A^{20}+\alpha A^{19}+\beta A = \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+\alpha 2^{19}+2\beta & 0 \\ 3\alpha+3\beta & 0 & 1-\alpha-\beta \end{bmatrix}$$

$$\text{RHS} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \alpha+\beta=0 \text{ and}$$

$2^{20}+\alpha 2^{19}+2\beta=4$
$2^{20}+\alpha(2^{19}-2)=4$

$$2 = \frac{4-2^{20}}{2^{19}-2} = -2$$

$\beta=2 \quad \Rightarrow(\alpha-\beta)=4$

Q.12 (3)

$$A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \quad \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A^4 = 2^2 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^8 = 64 \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} = 128 \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\Rightarrow 128 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\Rightarrow 128 \begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\Rightarrow \quad x-y = \frac{1}{16} \qquad .....(1)$$

$$\& \quad -x+y = \frac{1}{2} \qquad .....(2)$$

$\Rightarrow$ From (1) & (2) : No solution.

Q.13 [766]

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

diagonal elements of
AA^T, $a^2 + b^2 + c^2 + d^2 + e^2 + f^2$, $g^2 + b^2 + c^2 2$
Sum = $a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$
a, b, c, d, e, f, g, h, i $\in$ {0, 1, 2, 3}

	Case	No. of Matrices
(1)	All ? 1s	$\frac{9!}{9!}=1$
(2)	One $\rightarrow$ 3 remaining-0	$\frac{9!}{1!\times 8!}=9$
(3)	One-2 five-1s three-0s	$\frac{9!}{1!\times 5!\times 3!}=8\times 63$
(4)	two ? 2's one-1 six-0's	$\frac{9!}{2!\times 6!}=63\times 4$

Total no. of ways = 1 + 9 + 8 × 63 + 63 × 4

$= \boxed{766}$

Q.14 [36]

Let M = $(P^{-1}AP - I)^2$
$= (P^{-1}AP)^2 - 2P^{-1}AP + I$
$= P^{-1}A^2P - 2P^{-1}AP + I$
$PM = A^2P - 2AP + P$
$= (A^2 - 2A.I + I^2)P$
$\Rightarrow \text{Det}(PM) = \text{Det}((A - I)^2 \times P)$
$\Rightarrow \text{DetP.DetM} = \text{Det}(A - I)^2 \times \text{Det}(P)$
$\Rightarrow \text{Det } M = (\text{Det}(A - I))^2$

$$\text{Now } A-i = \begin{bmatrix} 1 & 7 & w^2 \\ -1 & -w-1 & 1 \\ 0 & -w & -w \end{bmatrix}$$

$\text{Det}(A - I) = (w^2 + w + w) + 7(-w) + w^3 = -6w$
$\text{Det}((A - I))^2 = 36w^2$
$\Rightarrow \alpha = 36$

Q.15 [1]

A = XB

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{3}\,a_1 \\ \sqrt{3}\,a_2 \end{bmatrix} = \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$$

$b_1 - b_2 = \sqrt{3}\,a_1 \qquad ...(1)$

$b_1 + kb_2 = \sqrt{3}\,a_2 \quad ...(2)$

Given, $a1^2 + a2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$

$(1)^2 + (2)^2$
$(b_1 + b^2)^2 + (b_1 + kb^2)^2 = 3(a_1^{\,2} + a_2^{\,2})^2$

$$a_1^{\,2} + a_2^{\,2} = \frac{2}{3}b_1^2 + \frac{(1+k^2)}{3}b_2^2 + \frac{2}{3}b_1 b_2(k-1)$$

Given, $a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{2}{3}b_2^2$

On comparing we get

$$\frac{k^2+1}{3} = \frac{2}{3} \Rightarrow \quad k^2+1=2$$

$\Rightarrow k = \pm 1$ ....(3)

& $\frac{2}{3}(k-1)\ 0 \Rightarrow k = 1$ ....(4)

From both we get k = 1

Q.16 (4)

$kx + y + z = 1$

$x + ky + z = k$

$x + y + zk = k^2$

$$\Delta = \begin{vmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{vmatrix} = K(K^2-1)-1(K-1)+1(1-K)$$

= K3 – K – K + 1 + 1 – K

= K3 – 3K + 2

= (K – 1)2 (K + 2)

For K = 1

$\Delta = D_1 = D_2 = D_3 = 0$

But for K = – 2, at least one out of D_1, D_2, D_3 are not zero

Hence for no sol^n, K = –2

Q.17 (3)

$A^2 = \sin^2 \alpha\ I$

So, $\left|A^2 - \frac{I}{2}\right| = \left(\sin^2 \alpha - \frac{1}{2}\right)^2 = 0$

$\Rightarrow \sin \alpha = \pm\frac{1}{\sqrt{2}}$

Q.18 [16]

2A adj (2A) = |2A|I

$\Rightarrow$ A adj (2A) = –4I....(i)

Now, E = $|A^4| + |A^{10} - (\text{adj}(2A))^{10}|$

$$= (-2)^4 + \frac{|A^{20} - A^{10}(\text{adj } 2A)^{10}|}{|A|^{10}}$$

$$= 16 + \frac{|A^{20} - (A\,\text{adj}(2A))^{10}|}{|A|^{10}}$$

$$= 16 + \frac{|A^{20} - 2^{10} I|}{2^{10}} \quad \text{(from (1))}$$

Now, characteristic roots of A are 2 and –1.

So, characteristic roots of A^{20} are 2^{10} and 1.

Hence, $(A^{20} - 2^{10} I)(A^{20} - I) = 0$

$\Rightarrow |A^{20} - 2^{10}I| = 0$ (as $A^{20} \neq I$)

$\Rightarrow$ E = 16 Ans.

Q.19 [2020]

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},\ B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$\Rightarrow$ AB = B

$\Rightarrow$ (A – I) B = O

$\Rightarrow$ |A – I | = O, since B ≠ O

$$\begin{vmatrix} (a-1) & b \\ c & (d-1) \end{vmatrix} = 0$$

ad – bc = 2020

Q.20 (2)

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$\Rightarrow -(a + b + g)(a^2 + b^2 + g^2 - \text{å}ab) = 0$

$\Rightarrow -(-a)(a^2 - 2b - b) = 0$

$\Rightarrow a(a^2 - 3b) = 0$

$\Rightarrow a^2 = 3b \Rightarrow \frac{a^2}{b} = 3$

Q.21 (2)

$$A + 2B = \begin{pmatrix} 1 & 2 & 0 \\ 6 & 3 & 3 \\ -5 & 3 & 1 \end{pmatrix} \quad ...(1)$$

$$2A - B = \begin{pmatrix} 2 & -1 & 5 \\ 2 & 1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\Rightarrow 4A - 2B = \begin{pmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{pmatrix} \quad ...(2)$$

$$(1) + (2) \Rightarrow 5A = \begin{pmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix} \text{ and } 2A = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix}$$

$$\therefore B = \begin{pmatrix} 2 & 0 & 4 \\ 4 & 2 & 6 \\ -2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 & 5 \\ 2 & 1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{pmatrix}$$

tr(A) = 1 – 1 + 1=1
tr(B)= –1
tr(A) = 1 and tr(B) = –1
$\therefore$ tr(A) – tr(B) = 2

Q.22 (1)
For non-trivial solution

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2\mu - 6\lambda + \lambda\mu = 12$
when $\mu = 6$, $12 - 6\lambda + 6\lambda = 12$
which is satisfied by all λ

Q.23 [6]

$$P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

$$5I8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n$$

$\Rightarrow n = 6$

Q.24 (4)

Q.25 (3)

Q.26 [3125]

Q.27 [11]
Q.28 (1)

Q.29 (4)

Q.30 (4)

Q.31 [2020]

Q.32 (1)

Q.33 (4)

Q.34 [910]

Q.35 (1)

Q.36 [*]

Q.37 (3)

Q.38 [8]

Q.39 (1)

Q.40 (2)

Q.41 (4)

Q.42 (4)

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 (C)
(In JEE this question was bonus because in JEE instead of 2n × 2n, 3 × 3 was given and we know that there is no non-singular 3×3 skew symmetric matrix).
Data inconsistent
A 3 × 3 non-singular matrix cannot be skew-symmetric
However considering M, N matrices as even order, we obtain correct answer.
$M^2 N^2 (M^T N)^{-1} (MN^{-1})^T = M^2N^2 N^{-1} (M^T)^{-1} (N^{-1})^T M^T$
$\Rightarrow \; - M^2 N^2 N^{-1} M^{-1} N^{-1} M$
$\Rightarrow \; - M^2 NM^{-1} N^{-1} M \qquad \Rightarrow - MNN^{-1} M$
$\Rightarrow \; -M^2$

Comprehension # 1 (Q. No. 2 to 4)

Q.2 (D)

Q.3 (A)

Q.4 (B)
$a + 8b + 7c = 0$ .... (i)
$9a + 2b + 3c = 0$ .... (ii)
$a + b + c = 0$ .... (iii)

$$\Delta = \begin{vmatrix} 1 & 8 & 7 \\ 9 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1.(-1) - 8(6) + 7(7) = 0$$

Let $c = \lambda$
$\therefore\ a + 8b = -7\lambda \qquad \Rightarrow\ a + b = -\lambda$

$$\Rightarrow\ b = \frac{-6}{7}\lambda \ \ \&\ \ a = \frac{-\lambda}{7}$$

$$\therefore\ (a, b, c) \equiv \left(\frac{-\lambda}{7}, \frac{-6\lambda}{7}, \lambda\right) \text{ where } \lambda \in R$$

2 P(a, b, c) lies on the plane 2x + y + z = 1

$\therefore \frac{-2\lambda}{7}-\frac{6\lambda}{7}+\lambda = 1 \qquad \Rightarrow \frac{-\lambda}{7} = 1$

$\Rightarrow \lambda = -7$

$\therefore$ 7a + b +c = 7 + 6 – 7 = 6

3 a = 2 $\qquad \Rightarrow \lambda = -14$

$\therefore$ b = 12 & c = –14

Now $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c} = \frac{3}{\omega^2}+\frac{1}{\omega^{12}}+3.\omega^{14} = 3\omega + 1 +$

$3\omega^2 = 3(\omega + \omega^2) + 1 = -2$

4 b = 6 $\Rightarrow \lambda = -7$

$\Rightarrow$ a = 1 & c = – 7

now $ax^2 + bx + c = 0 \qquad \Rightarrow x^2 + 6x - 7 = 0$

$\Rightarrow$ x = –7 , 1

$$\therefore \sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n = \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n = 1 + \frac{6}{7}+\left(\frac{6}{7}\right)^2 +$$

$$\text{.......} \infty = \frac{1}{1-\frac{6}{7}} = 7$$

Q.5 (A)

a, b, c $\in \{\omega, \omega^2\}$

Let A = $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$

$\Rightarrow \quad |A| = 1 - (a + c)\,\omega + ac\omega^2$

Now

|A| will be non-zero only when a = c = ω

$\therefore \quad (a, b, c) \equiv (\omega , \omega , \omega)$ or $(\omega , \omega^2, \omega)$

$\therefore$ number of non singular matrices = 2

Q.6 [9]

Let M = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then

$a_{12} = -1,$ $\qquad a_{11} - a_{12} = 1$

$\Rightarrow a_{11} = 0,$ $\qquad a_{11} + a_{12} + a_{13} = 0$

$\Rightarrow a_{13} = 1$ $\qquad a_{22} = 2$,

$a_{21} - a_{22} = 1$

$\Rightarrow a_{21} = 3,$ $\qquad a_{21} + a_{22} + a_{23} = 0$

$\Rightarrow a_{23} = -5$ $\qquad a_{32} = 3$,

$a_{31} - a_{32} = -1$

$\Rightarrow a_{31} = 2,$ $\qquad a_{31} + a_{32} + a_{33} = 12$

$\Rightarrow a_{33} = 7$

Hence sum of diagonal of M is = $a_{11} + a_{22} + a_{33} = 0 + 2 + 7 = 9$

Q.7 (D)

Given

$P = [a_{ij}]_{3\times3}$ $\qquad b_{ij} = 2^{i+j}$ aij

$Q = [b_{ij}]_{3\times3}$

$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ |P| = 2 ;

$$Q = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{bmatrix}$$

$$\text{Determinant of Q} = \begin{vmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{vmatrix} = 4$$

$$\times 8 \times 16 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 4a_{31} & 4a_{32} & 4a_{33} \end{vmatrix}$$

$$= 4 \times 8 \times 16 \times 2 \times 4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2^2 . 2^3 . 2^4 .$$

$2^1 . 2^2 . 2^1 = 2^{13}$

Q.8 (D)

$P^T = 2P + I$

$\Rightarrow \quad (P^T)^T = (2P + I)^T \qquad \Rightarrow \quad P = 2P^T + I$

$\Rightarrow \quad P = 2(2P + I) + I \qquad \Rightarrow \quad 3P = -3I$

$\Rightarrow P = -I$

$\Rightarrow \quad PX = -IX = -X$

Q.9 (AD)

Let $A = [a_{ij}]_{3\times3}$; $\qquad$ adj A = $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

|adj A| = 1(3 – 7) – 4(6 – 7) + 4(2 – 1) = 4

$\Rightarrow |A|^{3-1} = 4 \Rightarrow |A|^2 = 4 \Rightarrow |A| = \pm 2$

Q.10 (C,D)

(A) $(N^TMN)^T = N^T M^T N$ is symmetric if M is symmetric and skew-symmetric if M is skew-symmetric.

(B) $(MN - NM)^T = (MN)^T - (NM)^T = NM - MN = -(MN - NM)$ skew symmetric

(C) $(MN)^T = N^T M^T = NM \neq MN$ hence NOT correct

(D) standard result is adj(MN) = (adjN) (adj M)] ≠ (adjM) (adjN)

Q.11 (CD)

M = $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

(A) $\begin{bmatrix} a \\ b \end{bmatrix}$ & [b c] are transpose.

So $\begin{bmatrix} a \\ b \end{bmatrix}$ = $\begin{bmatrix} b \\ c \end{bmatrix}$ is given

⇒ a = b = c

M= $\begin{bmatrix} a & a \\ a & a \end{bmatrix}$

⇒ | M | = 0

A is wrong.

(B) [b c] & $\begin{bmatrix} a \\ b \end{bmatrix}$ are transpose.

So a = b = c

B is wrong

(C) M = $\begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}$ ⇒ | M | = ac ≠ 0

C is correct

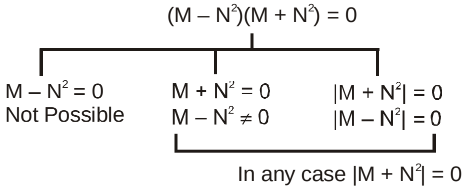
(D) M = $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$ given ac ≠ λ^2.

D is correct

(C, D) are correct.

Q.12 (AB)

MN = NM & $M^2 - N^4 = 0$



(A) $|M^2 + MN^2| = |M|\,|M + N^2|$

= 0 (A) is correct

(B) If |A| = 0 then AU = 0 will have ∞ solution.

Thus $(M^2 + MN^2)$ U = 0 will have many 'U'

(B) is correct

(C) Obvious wrong.

(D) If AX = 0 & |A| = 0 then X can be non zero.

(D) is wrong

Q.13 (C,D)

(C) $(x^4 Z^3 - Z^3 X^4)^T = (X^4Z^3)^T (Z^3 X^4)^T$

$= (Z^T)^3(X^T)^4 - (X^T)^4(Z^T)^3$

$= Z^3X^4 - X^4Z^3$

$= -(X^4Z^3 - Z^3X^4)$

(D) $(X^{23} + Y^{23})^T = -X^{23} - Y^{23} \Rightarrow X^{23} + Y^{23}$ is skew–symmetric

Q.14 (B, C)

$$|P| = 12\alpha + 20$$

$$\text{adj } P = \begin{bmatrix} 5\alpha & -10 & -\alpha \\ 3\alpha & -6 & -(3\alpha+4) \\ 10 & 12 & 2 \end{bmatrix}$$

$$\because \frac{Q}{k} = \frac{adj\,P}{|P|} \qquad \Rightarrow \qquad Q = \frac{k}{|P|} adj\ P$$

$$\because q_{23} = -\frac{k}{8} \qquad \Rightarrow \frac{(3\alpha+4)k}{(12\alpha+20)} = \frac{k}{8} \qquad \Rightarrow \alpha = -1$$

Also $|Q| = \dfrac{k^3}{|P|}$ $\Rightarrow k = 4$

Hence, (b, c)

Q.15 (B)

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} \Rightarrow P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2\times4 & 1 & 0 \\ (1+2)16 & 2\times4 & 1 \end{bmatrix}$$

$$\Rightarrow P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3\times4 & 1 & 0 \\ (1+2+3) & 3\times4 & 1 \end{bmatrix}$$

$$P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 50\times4 & 1 & 0 \\ (1+2+3+....+50)16 & 50\times4 & 1 \end{bmatrix}$$

$$\Rightarrow P^{15} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 20400 & 200 & 1 \end{bmatrix}$$

Now, $P^{50} - Q = I$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 20400 & 200 & 1 \end{bmatrix} - \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$20400 - q_{31} = 0 \Rightarrow q_{31} = 20400, 200 - q_{32} = 0$$

$$\Rightarrow q_{32} = 200, 200 - q_{21} = 0 \Rightarrow q_{21} = 200$$

$$\therefore \frac{q_{31}+q_{32}}{q_{21}} = \frac{20600}{200} = 103$$

Q.16 (A,C)

$A = B^2 \Rightarrow |A| = |B| = +\text{ ve}$

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = 1\,(-1) = \text{negative}$

Matrix B can not be possible

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Matrix B can be possible

Ex. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = -1 = \text{negative}$

Matrix B can not be possible

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 1 = \text{possible}$

Matrix B can be I

Q.17 [1]

D = 0

$$\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} \left(1+\alpha+\alpha^2\right) & (2\alpha+1) & \left(\alpha^2+\alpha+1\right) \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} \left(1+\alpha+\alpha^2\right) & (2\alpha+1) & 0 \\ \alpha & 1 & 0 \\ \alpha^2 & \alpha & 1-\alpha^2 \end{vmatrix} = 0 \Rightarrow (1-\alpha^2)$$

$(1 + \alpha + \alpha^2 - 2\alpha^2 - \alpha) = 0 \Rightarrow (1 - \alpha^2) = 0$

$\alpha = -1$ or 1

for $\alpha = 1$, system of linear equations has no solution

$\therefore \quad \alpha = -1$ so $1 + \alpha + \alpha^2 = 1$

Q.18 (A)

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 5$

Case-I : Five (1's) and four (0,s)

$\quad {}^9C_5 = 126$

Case-II : One (2) and one (1)

$\quad {}^9C_5 \times 2! = 72$

$\quad \therefore$ Total = 198

Q.19 (A, C, D)

We find D = 0 & since no pair of planes are parallel, so there are infinite number of solutions.

Let $\alpha P_1 + \lambda P_2 = P_3$

$\Rightarrow P_1 + 7P_2 = 13P_3$

$\Rightarrow b_1 + 7b_2 = 13b_3$

(A) $D \neq 0 \Rightarrow$ unique solution of any b_1, b_2, b_3

(B) D = 0 but $P_1 + 7P_2 \neq 13P_3$

(C) D = 0 Also $b_2 = -2b_1$, $b_3 = -b_1$

$\quad$ Satisfies $b_1 + 7b_2 = 13b_3$ (Actually all three planes are co-incident)

(D) $D \neq 0$

Q.20 (B)

Given $M = \alpha l + \beta M^{-1} \quad \Rightarrow M^2 - \alpha M - \beta I = O$

By putting values of M and M^2, we get

$\alpha(\theta) = 1 - 2\sin^2\theta \cos^2\theta = 1 - \dfrac{\sin^2 2\theta}{2} \geq \dfrac{1}{2}$

Also, $\beta(\theta) = -\left(\sin^4\theta\cos^4\theta + (1 + \cos^2\theta)(1 + \sin^2\theta)\right)$

$\quad = -\left(\sin^4\theta\cos^4\theta + 1 + \cos^2\theta + \sin^2\theta + \sin^2\theta\cos^2\theta\right)$

$\quad = -(t^2 - t + 2)$, $t = \dfrac{\sin^2 2\theta}{4} \in \left[0, \dfrac{1}{4}\right]$

$\Rightarrow \beta(\theta) \geq -\dfrac{37}{16}$

Q.21 (A, C, D)

$(\text{adj}M)_{11} = 2 - 3b = -1 \Rightarrow b = 1$

Also, $(\text{adj}M)_{22} = -3a = -6 \Rightarrow a = 2$

Now, det M = $\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = -2$

$\Rightarrow \det(\text{adj}M^2) = (\det M^2)^2$

$= (\det M)^4 = 16$

Also $M^{-1} = \dfrac{\text{adj}M}{\det M}$

$\Rightarrow \text{adj}M = -2M^{-1}$

$\Rightarrow (\text{adj}M)^{-1} \qquad = \dfrac{-1}{2}M$

And, $(\text{adj}M)^{-1} = (M^{-1})^{-1}\det(M^{-1})$

$= \dfrac{1}{\det M}M = \dfrac{-M}{2}$

Hence, $(\text{adj}M)^{-1} + \text{adj}(M^{-1}) = -M$

Further, $MX = b$

$$\Rightarrow X = M^{-1}b = \frac{-\text{adj}M}{2}b$$

$$= -\frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \frac{-1}{2}\begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$\Rightarrow (\alpha, \beta, \gamma) = (1, -1, 1)$

Q.22 (B,C,D)

$$\text{Let } Q = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

$$X = \sum_{k=1}^{6}\left(P_k Q P_K^T\right)$$

$$X^T = \sum_{k=1}^{6}\left(P_k Q P_K^T\right)^T = X.$$

X is symmetric

$$\text{Let } R = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$XR = \sum_{k=1}^{6} P_k Q P_K^T R.$$

$\left[\because P_K{}^T R = R\right]$

$$= \sum_{K=1}^{6} P_k QR. = \left(\sum_{k=1}^{6} P_K\right) QR$$

$$\sum_{k=1}^{6} P_K = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \qquad QR = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$$

$$\Rightarrow XR = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix} = 30R \;\Rightarrow \alpha = 30.$$

$$\text{Trace } X = \text{Trace}\left(\sum_{k=1}^{6} P_K Q P_K^T\right) = \sum_{k=1}^{6}\text{Trace}$$

$(P_K Q P_K{}^T) = 6\,(\text{Trace } Q) = 18$

$$X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 30\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow (X - 30\,I)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = O \Rightarrow |X - 30\,I| = 0$$

$\Rightarrow X - 30\,I$ is non-invertible

Q.23 (C,D)

$$\det(R) = \det(PQP^{-1}) = (\det P)(\det Q)\left(\frac{1}{\det P}\right)$$

$= \det Q$

$= 48 - 4x^2$

Option - 1 :

for $x = 1$ det $(R) = 44 \neq 0$

$$\therefore \text{ for equation } R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

We will have trivial solution

$\alpha = \beta = \gamma = 0$

Option - 2 :

$PQ = QP$

$PQP^{-1} = Q$

$R = Q$

No value of x.

Option - 3 :

$$\det\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$$

$= (40 - 4x^2) + 8 = 48 - 4x^2 = \det R \;\forall\; x \in R$

Option - 4 :

$$R = \begin{bmatrix} 2 & 1 & 2/3 \\ 0 & 4 & 4/3 \\ 0 & 0 & 6 \end{bmatrix}$$

$$(R - 61)\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = O$$

$$\Rightarrow -4 + a + \frac{4b}{3} = 0$$

$\Rightarrow a = 2$ $\qquad$ $b = 3$

$a + b = 5$

Q.24 (B,C,D)

det (M) $\neq 0$

$M^{-1} = \text{adj}(\text{adj } M)$

$M^{-1} = \det(M).M$

$M^{-1}M = \det(M).M^2$

$I = \det(M).M^2$ $\qquad$ ....(i)

$\det(I) = (\det(M))^5$

$I = \det(M)$ $\qquad$ ....(ii)

From (i) $I = M^2$

$(\text{adj } M^2) = \text{adj } (M^2) = \text{adj } I = I$

Q.25 [5]

M-I

$$\text{Let} A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^2 = \begin{bmatrix} a^2+bc & ab+bc \\ ac+dc & bc+d^2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a^3+2abc+bdc & a^2b+abd+b^2c+bd^2 \\ a^2c+adc+bc^2+d^2c & abc+2bcd+d^3 \end{bmatrix}$$

Given trace (A) $= a+d = 3$

and trace $(A^3) = a^3+d^3+3abc+3bcd = -18$

$\Rightarrow a^3+d^3+3bc(a+d) = -18$

$\Rightarrow a^3+d^3+9bc = -18$

$\Rightarrow (a+d)((a+d)^2-3ad)+9bc = -18$

$\Rightarrow 3(9-3ad)+9bc = -18$

$\Rightarrow ad-bc = 5 =$ determinant of A

M-II

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \qquad \Delta = ad-bc$$

$|A-\lambda I| = (a-\lambda)(d-\lambda)-bc$

$= \lambda^2-(a+d)\lambda+ad-bc$

$= \lambda^2-3\lambda+\Delta$

$\Rightarrow O = A^2-3A+\Delta I$

$\Rightarrow A^2 = 3A-\Delta I$

$\Rightarrow A^3 = 3A^2-\Delta A$

$= 3(3A-\Delta I)-\Delta A$

$= (9-\Delta)A-3\Delta I$

$$= (9-\Delta)\begin{bmatrix} a & b \\ c & d \end{bmatrix} - 3\Delta\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ trace $A^3 = (9-\Delta)(a+d)-6\Delta$

$\Rightarrow$ $-18 = (9-\Delta)(3)-6\Delta$

$= 27-9\Delta$

$\Rightarrow$ $9\Delta = 45$ $\qquad$ $\Rightarrow \Delta = 5$

Q.26 (A,B,D)

$$PEP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 8 & 13 & 18 \\ 2 & 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(B) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

$|E| = 0$ and $|F| = 0$ and $|Q| \neq 0$

$$|EQ| = |E||Q| = 0,\ |PFQ^{-1}| = \frac{|P||F|}{|Q|} = 0$$

$T = EQ + PFQ^{-1}$

$TQ = EQ^2 + PF = EQ^2 + P^2EP = EQ^2 + EP = E(Q^2 + P)$

$|TQ| = |E(Q^2 + P)| \Rightarrow |T||Q| = |E||Q^2 + P| = 0 \Rightarrow |T| = 0$

(as $|Q| \neq 0$)

(C) $|(EF)^3| > |EF|^2$

Here $0 > 0$ (false)

(D) as $P^2 = I \Rightarrow P^{-1} = P$ so $P^{-1}FP = PFP = PPEPP = E$

so $E + P^{-1}FP = E + E = 2E$

$P^{-1}EP + F \Rightarrow PEP + F = 2PEP$

$\text{Tr}(2PEP) = 2\text{Tr}(PEP) = 2\text{Tr}(EPP) = 2\text{Tr}(E)$

EXERCISES

ELEMENTRY

Q.1 (3)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}, \text{by } \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)\ (b-c)\begin{vmatrix} 0 & 0 & a-c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} \text{ by } R_1 \to R_1 - R_2$$

$$= (a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(a-c).(-1) = (a-b)(b-c)(c-a)$$

Q.2 (1)

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 4 & 3 & 6 \end{vmatrix} \text{by } C_1 \to C_1 + C_2$$

$$= \begin{vmatrix} 1 & 2 & 1 \\ 1 & 5 & 3 \\ 1 & 9 & 6 \end{vmatrix} \text{ by } C_2 \to C_2 + C_3$$

$$= \begin{vmatrix} 3 & 1 & 1 \\ 6 & 2 & 3 \\ 10 & 3 & 6 \end{vmatrix} \text{ by } C_1 \to C_1 + C_2 + C_3$$

$$\text{But} \neq \begin{vmatrix} 2 & 1 & 1 \\ 2 & 2 & 3 \\ 2 & 3 & 6 \end{vmatrix}$$

Q.3 (2)

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix}$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = 0$$

Q.4 (4)

by $C_1 \to C_1 + C_2 + C_3$

$$\text{we have } (9+x)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow (x+9)\begin{vmatrix} 0 & 1-x & 0 \\ 0 & -(1-x) & 1-x \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

Q.5 (4)

$$\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix} = \begin{vmatrix} a+b & a+2b & a+3b \\ b & b & b \\ 2b & 2b & 2b \end{vmatrix} = 0$$

$$\left\{\text{by } \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_2 \end{matrix}\right\}$$

Trick: Putting $a = 1 = b$. The determinant will be

$$\begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix} = 0$$. Obviously answer is (d)

Note : Students remember while taking the values of $a, b, c, \ldots\ldots$ that for there values, the options (a), (b), (c) and (d) should not be identical.

Q.6 (2)

The cofactor of element 4, in the 2nd row and 3rd column is

$$= (-1)^{2+3}\begin{vmatrix}1 & 3 & 1\\ 8 & 0 & 1\\ 0 & 2 & 1\end{vmatrix} = -\{1(-2) - 3(8-0) + 1.16\} =$$

10.

Q.7 (2)

(2) We know that

$$\Delta\Delta' = \begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix} . \begin{vmatrix}A_1 & B_1 & C_1\\ A_2 & B_2 & C_2\\ A_3 & B_3 & C_3\end{vmatrix}$$

$$= \begin{vmatrix}\Sigma a_1A_1 & 0 & 0\\ 0 & \Sigma a_2A_2 & 0\\ 0 & 0 & \Sigma a_3A_3\end{vmatrix} = \begin{vmatrix}\Delta & 0 & 0\\ 0 & \Delta & 0\\ 0 & 0 & \Delta\end{vmatrix} = \Delta^3$$

$\Rightarrow\ \Delta' = \Delta^2$

Q.8 (3)

$C_{21} = (-1)^{2+1}(18+21) = -39$

$C_{22} = (-1)^{2+2}(15+12) = 27$

$C_{23} = (-1)^{2+3}(-35+24) = 11$

Q.9 (2)

Minor of - 4 = $\begin{vmatrix}-2 & 3\\ 8 & 9\end{vmatrix} = -42$, 9 = $\begin{vmatrix}-1 & -2\\ -4 & -5\end{vmatrix} = -3$

and cofactor of - 4 = $(-1)^{2+1}(-42) = 42$,

cofactor of 9= $(-1)^{3+3}(-3) = -3$.

Q.10 (3)

(3) $\Delta = \begin{vmatrix}b^2+c^2 & a^2 & a^2\\ b^2 & c^2+a^2 & b^2\\ c^2 & c^2 & a^2+b^2\end{vmatrix}$

$$= -2\begin{vmatrix}0 & c^2 & b^2\\ b^2 & c^2+a^2 & b^2\\ c^2 & c^2 & a^2+b^2\end{vmatrix} \text{ ,by } R_1 \to R_1 - (R_2 + R_3)$$

$$= -2\begin{vmatrix}0 & c^2 & b^2\\ b^2 & a^2 & 0\\ c^2 & 0 & a^2\end{vmatrix} \text{ , by } \begin{matrix}R_2 \to R_2 - R_1\\ R_3 \to R_3 - R_1\end{matrix}$$

$= -2\{-c^2(b^2a^2) + b^2(-c^2a^2)\} = 4a^2b^2c^2$.

Trick: Put a = 1, b = 2, c = 3 so that the option give different values.

Q.11 (2)

$$\begin{vmatrix}y+z & x & y\\ z+x & z & x\\ x+y & y & z\end{vmatrix} = (x+y+z)\begin{vmatrix}2 & 1 & 1\\ z+x & z & x\\ x+y & y & z\end{vmatrix}$$

by $R_1 \to R_1 + R_2 + R_3$

$$= (x+y+z)\begin{vmatrix}1 & 1 & 1\\ x & z & x\\ x & y & z\end{vmatrix} \text{ ; by } C_1 \to C_1 - C_2$$

$= (x+y+z).\{(z^2 - xy) - (xz - x^2) + (xy - xz)\}$

$= (x+y+z)(x-z)^2 \Rightarrow k = 1$.

Trick : Put $x = 1, y = 2$, $z = 3$, then

$$\begin{vmatrix}5 & 1 & 2\\ 4 & 3 & 1\\ 3 & 2 & 3\end{vmatrix} = 5(7) - 1(12-3) + 2(8-9)$$

$= 35 - 9 - 2 = 24$

and $(x+y+z)(x-z)^2 = (6)(-2)^2 = 24$

$\therefore k = \frac{24}{24} = 1$

Q.12 (2)

$$\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix} = \lambda$$

Applying $C_2 \to C_2 - C_1$, and $C_3 \to C_3 - C_1$,

$$\begin{vmatrix}1+a & -a & -a\\ 1 & b & 0\\ 1 & 0 & c\end{vmatrix}$$

On expanding w.r.t. R_3 ,

$ab+bc+ca+abc=\lambda$

…….(i)

Given, $a^{-1}+b^{-1}+c^{-1}=0$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0 \Rightarrow ab+bc+ca=0$

$\Rightarrow \lambda = abc$, (From equation (i)).

Q.13 (2)

Applying $C_1 \to C_1+C_2+C_3$

$$f(x)=\begin{vmatrix}1 & (1+b^2)x & (1+c^2)x\\ 1 & 1+b^2x & (1+c^2)x\\ 1 & (1+b^2)x & 1+c^2x\end{vmatrix},$$

$(\because a^2+b^2+c^2+2=0)$

[Applying $R_2 \to R_2-R_1,\ R_3 \to R_3-R_1$]

$$=\begin{vmatrix}1 & (1+b^2)x & (1+c^2)x\\ 0 & 1-x & 0\\ 0 & 0 & 1-x\end{vmatrix}=(1-x)^2.$$

Hence degree of f(x) = 2.

Q.14 (4)

$$\begin{vmatrix}4+x^2 & -6 & -2\\ -6 & 9+x^2 & 3\\ -2 & 3 & 1+x^2\end{vmatrix}=x^4(14+x^2)$$

$=x.x^3(14+x^2)$

Hence, the determinant is divisible by x, x^3 and $(14+x^2)$, but not divisible by x^5 .

Q.15 (3)

$$\begin{vmatrix}0 & b^3-a^3 & c^3-a^3\\ a^3-b^3 & 0 & c^3-b^3\\ a^3-c^3 & b^3-c^3 & 0\end{vmatrix}$$

$$(b^3-a^3)(c^3-a^3)\begin{vmatrix}0 & 1 & 1\\ a^3-b^3 & 1 & 1\\ a^3-c^3 & 1 & 1\end{vmatrix}=0$$

$[C_2 \to C_2-C_1$ and $C_3 \to C_3-C_1]$ and then taking out common from IInd column (b^2-a^3) and (c^3-a^3) from IIIrd column].

Q.16 (3)

$$\begin{vmatrix}1+\sin^2\theta & \sin^2\theta & \sin^2\theta\\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta\\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta\end{vmatrix}=0$$

Using $C_1 \to C_1-C_2, C_2 \to C_2-C_3$

$$\Rightarrow \begin{vmatrix}1 & 0 & \sin^2\theta\\ -1 & 1 & \cos^2\theta\\ 0 & -1 & 1+4\sin 4\theta\end{vmatrix}=0$$

$\Rightarrow 2(1+2\sin 4\theta)=0 \Rightarrow \sin 4\theta=\frac{-1}{2}$

Q.17 (2)

$$f(x)=2(x-3)(x-5)\ ;\ \begin{vmatrix}1 & x+3 & 3(x^2+3x+9)\\ 1 & x+5 & 4(x^2+5x+25)\\ 1 & 1 & 3\end{vmatrix}$$

(Taking out $(x-3),(x-5)$ and 2 from Ist row, IInd row and IIrd column respectively)

$$f(x)=2(x-3)(x-5)\begin{vmatrix}0 & (x+2) & 3(x^2+3x+8)\\ 0 & 2 & x^2+11x+73\\ 1 & 1 & 3\end{vmatrix},$$

$(R_1 \to R_1-R_3$ and $R_2 \to R_2-R_1$)

$=2(x-3)(x-5)[1(x+2)$

$(x^2+11x+73)-6(x^2+3x+8)]$

$=2(x^2-8x+15)(x^3+13x^2+95x$

$+146-6x^2-18x-48)$

$=2(x^2-8x+15)(x^3+7x^2+77x+98)$

$=2(x^5-x^4+36x^3-413x^2+371x+1470)$

$f(1)=2928,\ f(3)=0,\ f(5)=0$

$\therefore f(1).f(3)+f(3).f(5)+f(5).f(1)=0+0+0$

$=0=f(3)$

Q.18 (4)

$$\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix} = \begin{vmatrix} y+z & x-z & x-y \\ 2y & 2x & 0 \\ 2z & 0 & 2x \end{vmatrix}$$

$R_2 \to R_2 + R_1$ and $R_3 \to R_3 + R_1$

$$= 4\begin{vmatrix} y+z & x-z & x-y \\ y & x & 0 \\ z & 0 & x \end{vmatrix}$$

$$= 4[(y+z)(x^2) - (x-z)(xy) \;\; +(x-y)(-zx)]$$

$$= 4[x^2y + zx^2 - x^2y + xyz - zx^2 + xyz] = 8xyz$$

Hence $k = 8$

Q.19 (3)

It has a non-zero solution if

$$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0 \Rightarrow -6k + 6 = 0 \Rightarrow k = 1$$

Q.20 (4)

Q.21 (1)

For the equation to be inconsistent $D = 0$

$$\therefore \quad D = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & k+3 \\ 2k+1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow k = -3 \qquad \text{and}$$

$$D_1 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \neq 0$$

So that system is inconsistent for $k = -3$.

Q.22 (4)

If the given system of equations has a non-trivial solution, then $\begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 29$.

Q.23 (1)

For unique solution of the given system $D \neq 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{vmatrix} \neq 0$$

So this depends on μ only.

Q.24 (1)

Given system of equation can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$$

On solving the above system we get the unique solution x = – 10, y = – 4, z = 16.

Q.25 (1)

$$\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1 + a(a^2) = 0 \Rightarrow a^3 = -1 \Rightarrow a = -1.$$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

$\Delta_1 = \Delta_0^2$

$\Delta_2 = \Delta_1^2 = \Delta_0^4$

$\Delta_3 = \Delta_2^2 = \Delta_0^8$

$\Delta_4 = \Delta_3^2 = \Delta_0^{16}$

and so on $\Delta_n = \Delta_0^{2^n}$ Ans.

Q.2 (4)

Clearly, $f(\theta) = 2\sin^2\theta - 1 = -\cos 2\theta$

$\therefore \quad f(\theta) = 0 \Rightarrow \cos 2\theta = 0$

$\Rightarrow \theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

So, number of solution are 4.

Q.3 (4)

We have $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\cos\theta & 1 \\ 1 & 1 & 1+\tan\theta \end{vmatrix} = 0$

Apply $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$.

We get $\begin{vmatrix} 1 & 1 & 1 \\ 0 & \cos\theta & 0 \\ 0 & 0 & \tan\theta \end{vmatrix} = 0$

$\therefore$ $\tan\theta \cdot \cos\theta = 0$ (on expanding along C_1)

But $\cos\theta = 0$ (Rejected)

$\Rightarrow$ $\tan\theta = 0$

$\Rightarrow$ $\theta = n\pi, n \in I$

$\theta = \pi, 2\pi, 3\pi.$

Q.4 (1)

$$\begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^3 & 1 \\ 3w^2 & 2w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} u^2 - v^2 & u^3 - v^3 & 0 \\ v^2 - w^2 & v^3 - w^3 & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} u + v & u^2 + v^2 + vu & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\Rightarrow \begin{vmatrix} u - w & (u^2 - w^2) + v(u - w) & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & u + w + v & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $(v^2 + w^2 + vw) - (v + w)\,[(v + w) + u] = 0$

$\Rightarrow$ $v^2 + w^2 + vw = (v + w)^2 + u\,(v + w)$

$\Rightarrow$ $uv + vw + wu = 0$ **Ans.**

Q.5 (3)

Consider the det. B, using $R_1 \to R_1 + R_2 + R_3$

$$B = 2\begin{vmatrix} a + p + x & b + q + y & c + r + z \\ a + x & b + y & c + z \\ a + p & b + q & c + r \end{vmatrix}$$

using $R_2 \to R_2 - R_1$ and

$R_3 \to R_3 - R_1$

$$= 2\begin{vmatrix} a + p + x & b + q + y & c + r + z \\ -p & -q & -r \\ -x & -y & -z \end{vmatrix}$$

using $R_1 \to R_1 + R_2 + R_3$

$B = 2$ det. $A = 2 \cdot 6 = 12$

Q.6 (1)

$$\begin{bmatrix} -1 & 2 & 1 \\ 3 + 2\sqrt{2} & 2 + 2\sqrt{2} & 1 \\ 3 - 2\sqrt{2} & 2 - 2\sqrt{2} & 1 \end{bmatrix}$$

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 4 + 2\sqrt{2} & 2\sqrt{2} & 0 \\ 4 - 2\sqrt{2} & -2\sqrt{2} & 0 \end{bmatrix} = 1(-8\sqrt{2} - 8 - 8\sqrt{2} + 8)$$

$= -16\sqrt{2}$

So absolute value is $16\sqrt{2}$

Q.7 (4)

α, β, γ are roots of $x^3 + px + q = 0$

$\therefore$ $\alpha + \beta + \gamma = 0$ Here $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$(\alpha + \beta + \gamma)\begin{vmatrix} 1 & \beta & \gamma \\ 1 & \gamma & \alpha \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

Q.8 (4)

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^y + b^{-y})^2 & (b^3 - b^{-y})^2 & 1 \\ (c^2 + c^{-2})^2 & (c^2 - c^{-2})^2 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$\begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (b^y - b^{-y})^2 & 1 \\ 4 & (c^z - c^{-z})^2 & 1 \end{vmatrix} = 0$$

Q.9 (2)

$$\begin{vmatrix} b_1 + c_1 & c_1 + a_1 & a_1 + b_1 \\ b_2 + c_2 & c_2 + c_2 & a_2 + b_2 \\ b_3 + c_3 & c_3 + a_3 & a_3 + b_3 \end{vmatrix}$$

Taking two common, applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a_1 + b_1 + c_1) & c_1 + a_1 & a_1 + b_1 \\ 2(a_2 + b_2 + c_2) & c_2 + a_2 & a_2 + b_2 \\ 2(a_3 + b_3 + c_3) & c_3 + a_3 & a_3 + b_3 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$= 2\begin{vmatrix} a_1+b_1+c_1 & -b_1 & -c_1 \\ a_2+b_2+c_2 & -b_2 & -c_2 \\ a_3+b_3+c_3 & -b_3 & -c_3 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= 2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Q.10 (3)

$$\Delta = \begin{vmatrix} x & x+y & x+y+z \\ 2x & 5x+2y & 7x+5y+2z \\ 3x & 7x+3y & 9x+7y+3z \end{vmatrix} = -16$$

Applying $R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 - 3R_1$

$$\Delta = \begin{vmatrix} x & x+y & x+y+z \\ 0 & 3x & 5x+3y \\ 0 & 4x & 6x+4y \end{vmatrix} = -16$$

Applying $R_3 \to R_3 - R_2$

$$\begin{vmatrix} x & x+y & x+y+z \\ 0 & 3x & 5x+3y \\ 0 & x & x+y \end{vmatrix} = -16$$

Applying $R_2 \to R_2 - 3R_1$

$$\begin{vmatrix} x & x+y & x+y+z \\ 0 & 0 & 2x \\ 0 & x & x+y \end{vmatrix} = -16 \Rightarrow -2x(x^2 - 0) = -$$

$16 \Rightarrow x^3 = 8 \Rightarrow x = 2$

Q.11 (2)

$$\begin{vmatrix} b_1+c_1 & c_1+a_1 & a_1+b_1 \\ b_2+c_2 & c_2+c_2 & a_2+b_2 \\ b_3+c_3 & c_3+a_3 & a_3+b_3 \end{vmatrix}$$

Taking two common, applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a_1+b_1+c_1) & c_1+a_1 & a_1+b_1 \\ 2(a_2+b_2+c_2) & c_2+a_2 & a_2+b_2 \\ 2(a_3+b_3+c_3) & c_3+a_3 & a_3+b_3 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$= 2\begin{vmatrix} a_1+b_1+c_1 & -b_1 & -c_1 \\ a_2+b_2+c_2 & -b_2 & -c_2 \\ a_3+b_3+c_3 & -b_3 & -c_3 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= 2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Q.12 (3)

$$\Delta = \begin{vmatrix} x & x+y & x+y+z \\ 2x & 5x+2y & 7x+5y+2z \\ 3x & 7x+3y & 9x+7y+3z \end{vmatrix} = -16$$

Applying $R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 - 3R_1$

$$\Delta = \begin{vmatrix} x & x+y & x+y+z \\ 0 & 3x & 5x+3y \\ 0 & 4x & 6x+4y \end{vmatrix} = -16$$

Applying $R_3 \to R_3 - R_2$

$$\begin{vmatrix} x & x+y & x+y+z \\ 0 & 3x & 5x+3y \\ 0 & x & x+y \end{vmatrix} = -16$$

Applying $R_2 \to R_2 - 3R_1$

$$\begin{vmatrix} x & x+y & x+y+z \\ 0 & 0 & 2x \\ 0 & x & x+y \end{vmatrix} = -16 \Rightarrow -2x(x^2 - 0) = -$$

$16 \Rightarrow x^3 = 8 \Rightarrow x = 2$

Q.13 (2)

$$\begin{vmatrix} \cos(\theta+\varphi) & -\sin(\theta+\varphi) & \cos 2\varphi \\ \sin\theta & \cos\theta & \sin\varphi \\ -\cos\theta & \sin\theta & \cos\varphi \end{vmatrix}$$

$$= \frac{1}{\sin\phi\cos\phi}\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi \\ \sin\theta\sin\phi & \sin\phi\cos\theta & \sin^2\phi \\ -\cos\theta\cos\phi & \sin\theta\cos\phi & \cos^2\phi \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$= \frac{1}{\sin\phi\cos\phi}\begin{vmatrix} 0 & 0 & 2\cos^2\phi \\ \sin\theta\sin\phi & \sin\phi\cos\theta & \sin^2\phi \\ -\cos\theta\cos\phi & \sin\theta\cos\phi & \cos^2\phi \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 2\cos^2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix} = 2\cos^2\phi\ (\sin^2\theta +$$

$\cos^2\theta) = 2\cos^2\phi$

Q.14 (2)

$$\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix};$$

$$\Delta = \sin^2\theta\cos\theta \begin{vmatrix} \cos\phi & \sin\phi & \cot\theta \\ \cos\phi & \sin\phi & -\tan\theta \\ -\sin\phi & \cos\phi & 0 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\Delta = \sin^2\theta\cos\theta \begin{vmatrix} 0 & 0 & \cot\theta+\tan\theta \\ \cos\phi & \sin\phi & -\tan\theta \\ -\sin\phi & \cos\phi & 0 \end{vmatrix}$$

$\Delta = \sin\theta$

Q.15 (2)

Expand the determinant using first row and use $x - y = A$, $y - z = B$ and $z - x = C$

$\Rightarrow A + B + C = 0$

Q.16 (1)

Δ =

$$\begin{vmatrix} 1+a^2+a^4 & a+ab+a^2b^2 & 1+ac+a^2c^2 \\ 1+ab+a^2b^2 & 1+b^2+b^4 & 1+bc+b^2c^2 \\ 1+ac+a^2c^2 & 1+bc+b^2c^2 & 1+c^2+c^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$= (a-b)^2 (b-c)^2 (c-a)^2$

Q.17 (4)

For non trivial solution $\begin{vmatrix} \sin\theta & -\cos\theta & \lambda+1 \\ \cos\theta & \sin\theta & -\lambda \\ \lambda & \lambda+1 & \cos\theta \end{vmatrix} = 0$

; this gives $2\cos\theta(\lambda^2+\lambda+1) = 0$

Q.18 (4)

For non trivial solution

$$\begin{vmatrix} 1 & -\cos\theta & \cos 2\theta \\ -\cos\theta & 1 & -\cos\theta \\ \cos 2\theta & -\cos\theta & 1 \end{vmatrix} = 0$$

using $C_1 \to C_1 - C_3$

$$\begin{vmatrix} 2\sin^2\theta & -\cos\theta & \cos 2\theta \\ 0 & 1 & -\cos\theta \\ -2\sin^2\theta & -\cos\theta & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 2\sin^2\theta \begin{vmatrix} 1 & -\cos\theta & \cos 2\theta \\ 0 & 1 & -\cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix} = 0$$

$\sin^2\theta = 0$

or $1[1 - \cos^2\theta] - 1[\cos^2\theta - \cos 2\theta]$

$\sin^2\theta - [\cos^2\theta - (\cos^2\theta - \sin^2\theta)]$

$\sin^2\theta - \sin^2\theta = 0$

hence $D = 0 \ \forall \ \theta \in R$

$\Rightarrow$ (4)

Q.19 (1)

For non-trivial solution

$$\begin{vmatrix} (\alpha+a) & \alpha & \alpha \\ \alpha & \alpha+b & \alpha \\ \alpha & \alpha & \alpha+c \end{vmatrix} = 0$$

Taking α as common from each row

$$\Rightarrow \alpha^3 \begin{vmatrix} 1+\frac{a}{\alpha} & 1 & 1 \\ 1 & 1+\frac{b}{\alpha} & 1 \\ 1 & 1 & 1+\frac{c}{\alpha} \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$ and expanding

$$\Rightarrow \alpha^3 \left[\frac{ab}{\alpha^2} + \frac{bc}{\alpha^2} + \frac{ac}{\alpha^2} + \frac{abc}{\alpha^3}\right] = 0$$

$$\Rightarrow \frac{1}{\alpha} = -\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

Q.20 (1)

$$\Delta = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix}$$

For no solution of system $\Delta = 0$ and at least one of the Δ_x, Δ_y, Δ_z is non zero.

for $\Delta = 0$, $\lambda = -2$

Q.21 (2)

$D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 3 & -3 \end{vmatrix}$ which vanishes

hence for atleast one solution $D_1 = D_2 = D_3 = 0$

$\therefore \quad D_1 = \begin{vmatrix} a & -2 & 1 \\ b & 1 & -2 \\ c & 3 & -3 \end{vmatrix} = 0 \Rightarrow a - b + c = 0$ **Ans.**

Q.22 (2)

For non zero solution

$D = 0$

$$D = \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$1(3bc - 4bc) - 2a\,(c - b) + a(4c - 3b) = 0$

$-bc - 2ac + 2ab + 4ac - 3ab = 0$

$-bc + 2ac - ab = 0$

$ab + bc = 2ac$

Divide both side by abc

$$\frac{1}{c} + \frac{1}{a} = \frac{2}{b}$$

Hence a, b, c are in H.P.

JEE-ADVANCED OBJECTIVE QUESTIONS

Q.1 (C)

$2\log_{10} a + \log_{10}(a - 1) = \log_{10} 2a$

$\therefore\ a^2(a - 1) = 2a \qquad a \neq 0$

$\therefore\ a^2 - a - 2 = 0 \ \Rightarrow \qquad (a - 2)(a + 1) = 0$

$\Rightarrow \qquad a = 2\,,\ -1 \qquad \therefore \qquad a = 2$]

Q.2 (B)

Q.3 (A)

$$\begin{bmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{bmatrix}$$

$$= \frac{1}{abc}\begin{bmatrix} a(a^2+1) & a^2b & a^2c \\ b^2a & b(b^2+1) & b^2c \\ c^2a & c^2b & c(c^2+1) \end{bmatrix}$$

$$= \frac{abc}{abc}\begin{bmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{bmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix} = (a^2 + b^2 + c^2 + 1)$$

Q.4 (A)

$$\begin{vmatrix} a^3-x & a^4-x & a^5-x \\ a^5-x & a^6-x & a^7-x \\ a^7-x & a^8-x & a^9-x \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} a^3-x & a^4-x & a^5-x \\ a^5-a^3 & a^6-a^4 & a^7-a^5 \\ a^7-a^5 & a^8-a^6 & a^9-a^7 \end{vmatrix} = a^3a^5$$

$$\begin{vmatrix} a^3-x & a^4-x & a^5-x \\ a^2-1 & a^3-a & a^4-a^2 \\ a^2-1 & a^3-a & a^4-a^2 \end{vmatrix} = 0$$

Q.5 (C)

$$\Delta_1 = \begin{vmatrix} 2a & b & e \\ 2d & e & f \\ 4x & 2y & 2z \end{vmatrix} = 4\begin{vmatrix} a & b & e \\ d & e & f \\ x & y & z \end{vmatrix};$$

$$\Delta_2 = \begin{vmatrix} f & 2d & e \\ 2z & 4x & 2y \\ e & 2a & b \end{vmatrix} = 4\begin{vmatrix} f & d & e \\ z & x & y \\ e & a & b \end{vmatrix}$$

$C_1 \leftrightarrow C_1$ followed by

$$\Delta_2 = \begin{vmatrix} d & e & f \\ x & y & z \\ a & b & c \end{vmatrix}$$

$R_1 \leftrightarrow R_2$ followed by $R_1 \leftrightarrow R_3$ $= \begin{vmatrix} a & b & e \\ d & e & f \\ x & y & z \end{vmatrix}$

$\Delta_2 = \Delta_1$

$\Rightarrow\ \Delta_1 - \Delta_2 = 0$

Q.6 (D)

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$(\sin x + 2\cos x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

(sin x + 2 cos x)

$$\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

$\Rightarrow$ (sin x + 2 cos x) (sin x – cos x)^2 = 0

Hence tan x = –2 or tan x = 1

Total 200 solutions.

Q.7 **(C)**

$C_1 \to C_1 + C_2 + C_3$

f (x)

$$= \begin{vmatrix} 1+2x+x(a^2+b^2+c^2) & (1+b^2)x & (1+c^2)x \\ 1+2x+x(a^2+b^2+c^2) & 1+b^2x & (1+c^2)x \\ 1+2x+x(a^2+b^2+c^2) & (1+b^2)x & 1+c^2x \end{vmatrix}$$

(as $a^2 + b^2 + c^2 = -2$)

$$\begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 1-x & 1-x \end{vmatrix}$$

f (x) = (1 – x)^2 = 1 – 2x + x^2 $\Rightarrow$ (C)]

Q.8 (B)

$$\begin{vmatrix} a^3+1 & a^2b & a^2c \\ ab^2 & b^3+1 & b^2c \\ ac^2 & bc^2 & c^3+1 \end{vmatrix} = 11$$

$$\Rightarrow \frac{1}{a{\cdot}b{\cdot}c}\begin{vmatrix} a(a^3+1) & a^3b & a^3c \\ ab^3 & b(b^3+1) & b^3c \\ ac^3 & bc^3 & c(c^3+1) \end{vmatrix} = 11$$

$$\Rightarrow \frac{a.b.c}{a.b.c}\begin{vmatrix} a^3+1 & a^3 & a^3 \\ b^3 & b^3+1 & b^3 \\ c^3 & c^3 & c^3+1 \end{vmatrix} = 11$$

Applying $c_1 \to c_1 - c_2$, $c_2 \to c_2 - c_3$, we get

$$\Rightarrow \begin{vmatrix} 1 & 0 & a^3 \\ -1 & 1 & b^3 \\ 0 & -1 & c^3+1 \end{vmatrix} = 11$$

$\Rightarrow$ $a^3 + b^3 + c^3 + 1 = 11$ $\Rightarrow$ $a^3 + b^3 + c^3 = 10$

Only possibles are (1, 1, 2) (1, 2, 1), (2, 1, 1)

Number of triplets = 3.

Q.9 (A)

[Hint : Use $C_2 \to C_2 - C_1 - 2\,C_3$ then $C_1 \to C_1 + C_2$ take $a^2 + b^2 + c^2$ common from first column]

Q.10 (A)

(i) $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & c \end{vmatrix} = 0$

Applying $C_3 \to C_3 - C_2$

$$\begin{vmatrix} a & b & ax \\ b & c & bx \\ ax+b & bx+c & -bx \end{vmatrix} = 0 \Rightarrow$$

$$x\begin{vmatrix} a & b & a \\ b & c & b \\ ax+b & bx+c & -b \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_3$

$$x\begin{vmatrix} 0 & b & a \\ 0 & c & b \\ ax+2b & bx+c & -b \end{vmatrix} \Rightarrow x(ax + 2b)(b^2 - ac) = 0$$

$\therefore$ Non zero root of equation $x = -\dfrac{2b}{a}$

(ii) $\begin{vmatrix} 15-2x & 11 & 10 \\ 11-3x & 17 & 16 \\ 7-x & 14 & 13 \end{vmatrix}$

Applying $\begin{vmatrix} 15-2x & 1 & 10 \\ 11-3x & 1 & 16 \\ 7-x & 1 & 13 \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_3$ & $R_2 \to R_2 - R_3$

$\begin{vmatrix} 8-x & 0 & -3 \\ 4-2x & 0 & 3 \\ 7-x & 1 & 13 \end{vmatrix} = 0 \Rightarrow -1[(8 - x)3 + 3(4 - 2x)] = 0 \Rightarrow 9x = 36 \Rightarrow x = 4$

Q.11 (A)

For non-trivial solution $\begin{vmatrix} (\alpha+a) & \alpha & \alpha \\ \alpha & \alpha+b & \alpha \\ \alpha & \alpha & \alpha+c \end{vmatrix} = 0$

Taking α as common from each row

$$\Rightarrow \alpha^3 \begin{vmatrix} 1+\frac{a}{\alpha} & 1 & 1 \\ 1 & 1+\frac{b}{\alpha} & 1 \\ 1 & 1 & 1+\frac{c}{\alpha} \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$ and expanding

$$\Rightarrow \alpha^3 \left[\frac{ab}{\alpha^2}+\frac{bc}{\alpha^2}+\frac{ac}{\alpha^2}+\frac{abc}{\alpha^3}\right] = 0 \qquad \Rightarrow \frac{1}{\alpha} = -\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Q.12 (A)

$$\Delta = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix}$$

For no solution of system $\Delta = 0$ and at least one of the Δ_x, Δ_y, Δ_z is non zero. for $\Delta = 0$, $\lambda = -2$

Q.13 (B)

Here $\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & -3 \\ 2 & 5 & -\lambda \end{vmatrix}$ system has unique solution if

$\Delta \neq 0$ and at least one of Δ_x, Δ_y, Δ_z is non-zero.

$\Delta = 1(-2\lambda + 15) - 1(-\lambda + 6) - 1(5 - 4) \neq 0 \Rightarrow -2\lambda + 15 + \lambda - 6 - 1 \neq 0 \Rightarrow -\lambda + 8 \neq 0 \Rightarrow \lambda \neq 8$

Q.14 (D)

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & p & 2 \\ 1 & 4 & \mu \end{vmatrix}; \qquad \Delta_x = \begin{vmatrix} 4 & 2 & 3 \\ 3 & p & 2 \\ 3 & 4 & \mu \end{vmatrix};$$

$$\Delta_y = \begin{vmatrix} 1 & 4 & 3 \\ 1 & 3 & 2 \\ 1 & 3 & \mu \end{vmatrix}; \qquad \Delta_z = \begin{vmatrix} 1 & 2 & 4 \\ 1 & p & 3 \\ 1 & 4 & 3 \end{vmatrix}$$

For infinite no. of solution $\Delta = \Delta_x = \Delta_y = \Delta_z = 0 \Rightarrow \mu = 2$, $p = 4$

Q.15 (B)

For non-trivial solution $\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$

$\Rightarrow \lambda = \sin 2\alpha + \cos 2\alpha \Rightarrow -\sqrt{2} \leq \sin 2\alpha + \cos 2\alpha \leq \sqrt{2} \Rightarrow -\sqrt{2} \leq \lambda \leq \sqrt{2}$.

Q.16 (D)

$(a - 1)\,x + 0y + z = \alpha$ ....(1)

$x + (b - 1)\,y + 0z = \beta$ ....(2)

$0x + y + (c - 1)\,z = \gamma$ ....(3)

For no unique solution D = 0

$$\begin{vmatrix} (a-1) & 0 & 1 \\ 1 & (b-1) & 0 \\ 0 & 1 & (c-1) \end{vmatrix} = 0$$

$(a - 1)(b - 1)(c - 1) + 1 = 0$

$\therefore$ a = 2; b = 2; c = 0

Hence, | a + b + c | = 4.

Q.17 (C)

To have a non-trivial solution, we must have

$$\begin{vmatrix} k & k+1 & k-1 \\ k+1 & k & k+2 \\ k-1 & k+2 & k \end{vmatrix} = 0 \Rightarrow 2k + 1 = 0$$

$$\Rightarrow \ k = \frac{-1}{2}.$$

Aliter : Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} -1 & 1 & -3 \\ 2 & -2 & 2 \\ k-1 & k+2 & k \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

we get, $\begin{vmatrix} -2 & 4 & -3 \\ 4 & -4 & 2 \\ -3 & 2 & k \end{vmatrix} = 0$

Expanding along R_1, we get

$-2(-4k - 4) - 4(4k + 6) - 3(8 - 12) = 0 \Rightarrow$

$8k + 8 - 16k - 24 - 24 + 36 = 0$

$\Rightarrow \quad -4 - 8k = 0 \Rightarrow 8k = -4$

$\therefore \quad k = \frac{-1}{2}$.

Q.18 (A)

$$\Delta = \begin{vmatrix} p+a & b & c \\ a & q+b & c \\ a & b & r+c \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

and expansion

$pqc + \big(q(p+a) + bp\big)\, r = 0$

$$\frac{c}{r} + 1 + \frac{a}{p} + \frac{b}{q} = 0.$$

JEE-ADVANCED
MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (AC)

$$f(x) = \begin{vmatrix} a^{-x} & e^{x\ell na} & x^2 \\ a^{-3x} & e^{3x\ell na} & x^4 \\ a^{-5x} & e^{5x\ell na} & 1 \end{vmatrix} = \begin{vmatrix} a^{-x} & a^{x} & x^2 \\ a^{-3x} & a^{3x} & x^4 \\ a^{-5x} & a^{5x} & 1 \end{vmatrix}$$

$$f(-x) = \begin{vmatrix} a^{-x} & e^{-x\ell na} & x^2 \\ a^{3x} & e^{-3n\ell na} & x^4 \\ a^{5x} & e^{-5x\ell na} & 1 \end{vmatrix} = \begin{vmatrix} a^{x} & a^{-x} & x^2 \\ a^{3x} & a^{-3x} & x^4 \\ a^{5x} & a^{-5x} & 1 \end{vmatrix}$$

$= -\, f(x)$

$\therefore\ f(x) + f(-x) = 0$

Q.2 (ABD)

$$(\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix},\ \Delta = (1 + x + x^2) \begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$$

$= (1 + x + x^2)\ \{1(1 - x^3) - x(1 - x) + x^2(x^2 - 1)\}$

$= (1 + x + x^2)\ \{1 - x^3 - x + x^2 + x^4 - x^2) = (1 + x + x^2)\ \{x^4 - x^3 - x + 1)$

$\Delta = (1 - x^3)^2$; $\Delta' = 2(1 - x^3)\ (-3\ x^2)$; $\Delta'(1) = 0$

Q.3 (AC)

$p = a$; $q = a + d$; $r = a + 2\,d$; $s = a + 3\,d \Rightarrow f(x) = -2\,d^2$

Also use $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

Q.4 (AB)

$$(\Delta = \begin{vmatrix} x & 2y - z & -z \\ y & 2x - z & -z \\ y & 2y - z & 2x - 2y - z \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} x & 2y - z & -z \\ y - x & 2(x - y) & 0 \\ y - x & 0 & 2(x - y) \end{vmatrix} = (x - y)^2$$

$$\begin{vmatrix} x & 2y - z & -z \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{vmatrix} = 4(x - y)^2\ (x + y - z)$$

So Δ is divisible by $(x - y)$ & $(x - y)^2$.

Q.5 (CD)

We have $D_1 = (a + b + c)(c - a)^2$

and $D_2 = (a - b)(b - c)(c - a)\ (a + b + c)(a^2 + b^2 + c^2)$

Q.6 (ABCD)

$$\begin{vmatrix} a_1 - b_1 & a_1 - b_2 & a_1 - b_3 \\ a_2 - b_1 & a_2 - b_2 & a_2 - b_3 \\ a_3 - b_1 & a_3 - b_2 & a_3 - b_3 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_2$

$$\begin{vmatrix} a_1 - b_1 & a_1 - b_2 & a_1 \\ a_2 - a_1 & a_2 - a_1 & a_2 - a_1 \\ a_3 - a_2 & a_3 - a_2 & a_3 - a_2 \end{vmatrix} =$$

$$\begin{vmatrix} a_1 - b_1 & a_1 - b_2 & a_1 \\ d & d & d \\ d & d & d \end{vmatrix} = 0$$

$\Delta = 0$

Q.7 (ABC)

$$(\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix} = (a + b - x) \begin{vmatrix} 1 & a & b \\ 1 & -x & a \\ 1 & b & -x \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (a + b - x) \begin{vmatrix} 1 & a & b \\ 0 & -(x + a) & a - b \\ 0 & b - a & -(x + b) \end{vmatrix} = (a + b - x)$$

$\{(x + a)\ (x + b) + (a - b)^2\}$

Q.8 (BD)

$$(\Delta = \begin{vmatrix} b & c & b\alpha + c \\ c & d & c\alpha + d \\ b\alpha + c & c\alpha + d & a\alpha^3 - c\alpha \end{vmatrix} = 0$$

Applying $C_3 \to C_3 - (C_1\alpha + C_2)$

$$\Delta = \begin{vmatrix} b & c & 0 \\ c & d & 0 \\ b\alpha + c & c\alpha + d & a\alpha^3 - b\alpha^2 - 3c\alpha - d \end{vmatrix} = 0$$

$(a\alpha^3 - b\alpha^2 - 3c\alpha - d)\ (bd - c^2) = 0$

$\therefore$ Either b, c, d in G.P. or α is root of $ax^3 - bx^2 - 3cx - d = 0$

Q.9 (AC)

$$(\Delta = \begin{vmatrix} a^2(1 + x) & ab & ac \\ ab & b^2(1 + x) & bc \\ ac & bc & c^2(1 + x) \end{vmatrix} = a^2b^2c^2$$

$$\begin{vmatrix} (1 + x) & 1 & 1 \\ 1 & (1 + x) & 1 \\ 1 & 1 & (1 + x) \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$a^2b^2c^2(3+x)\begin{vmatrix}1&1&1\\1&(1+x)&1\\1&1&(1+x)\end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$= \begin{vmatrix}0&-x&0\\0&x&-x\\1&1&1+x\end{vmatrix} a^2b^2c^2(3+x) = a^2b^2c^2(3+x)\,x^2$$

Which is divisible by x^2

Q.10 (ABD)

$$f(x) = \begin{vmatrix}1/x&\log x&x^n\\1&-1/n&(-1)^n\\1&a&a^2\end{vmatrix};$$

$$f'(x) = \begin{vmatrix}-1/x^2&1/x&nx^{n-1}\\1&-1/n&(-1)^n\\1&a&a^2\end{vmatrix}$$

$$f''(x) = \begin{vmatrix}2/x^3&-1/x^2&n(n-1)x^{n-2}\\1&-1/n&(-1)^n\\1&a&a^2\end{vmatrix};$$

$$f^n(x) = \begin{vmatrix}(-1)^n\dfrac{n!}{x^{n+1}}&\dfrac{(-1)^{n-1}(n-1)!}{x^n}&n!\\1&-1/n&(-1)^n\\1&a&a^2\end{vmatrix}$$

$$f^n(1) = \begin{vmatrix}(-1)^n n!&(-1)^{n-1}(n-1)!&n!\\1&-1/n&(-1)^n\\1&a&a^2\end{vmatrix} = (-1)^n\ n!$$

$$\begin{vmatrix}1&-1/n&(-1)^n\\1&-1/n&(-1)^n\\1&a&a^2\end{vmatrix} = 0$$

and $y = a\,(x - f^n(1))$
$y = ax$

Q.11 (BCD)

$$f(x) = \begin{vmatrix}2\sin x&\sin^2 x&0\\1&2\sin x&\sin^2 x\\0&1&2\sin x\end{vmatrix} = 2\sin x\,(4\sin^2 x$$

$- \sin^2 x) - \sin^2 x(2\sin x) = 6\sin^3 x - 2\sin^3 x$
$f(x) = 4\sin^3 x$
$\Rightarrow\ f'(x) = 12\sin^2 x\cos x$
(B) $f'(\pi/2) = 12\sin^2(\pi/2)\cos(\pi/2) = 0$

(C) $f(-x) = -f(x)$ odd function

$$\therefore\ \int_{-\pi/2}^{\pi/2} f(x)dx = 0$$

(D) at $x = 0$, $y = 0$

$$\Rightarrow\ \left(\frac{dy}{dx}\right)_{(0,0)} = 0 \text{ tangent at } (0, 0)$$

$$y - 0 = \left(\frac{dy}{dx}\right)_{(0,0)}(x - 0) \quad \Rightarrow\ y = 0$$

Q.12 (ABCD)

$$f(x) = \begin{vmatrix}2\cos x&1&0\\1&2\cos x&1\\0&1&2\cos x\end{vmatrix};$$

$f(x) = 8\cos^3 x - 2\cos x - 2\cos x = 2\cos 3x + 2\cos x$
$f'(x) = -6\sin 3x - 2\sin x = 0$
$\therefore\ f'(0) = 0$ (D) ;
$f''(x) = -18\cos 3x - 2\cos x < 0$, at $x = 0$
$\therefore\ f(0)_{max} = 2 + 2 = 4$;

$$\int_0^\pi f(x)dx = \int_0^\pi (2\cos 3x + 2\cos x)\,dx = 0$$

Q.13 (ABCD)
For n^th order determinant $\Delta = |C_{ij}| = D^{n-1}$
(A) For 3rd order determinant $\Delta = D^{3-1} = D^2$ ... (1)
(B) From (1) if $D = 0$ then $\Delta = 0$
(C) $\Delta = 27 = 3^3$
$\Delta = (3^3)^2 = 3^6$ (a perfect cube)

Q.14 (ABD)
(Det $(-A) = (-1)^n$ det (A) where n is order of square matrix.

Q.15 (A,D)

$$\begin{vmatrix}1+\beta^2&\beta&\beta^2\\3&\alpha-2&3\\\alpha&1&\alpha\end{vmatrix} = 0 = \begin{vmatrix}1&\beta&\beta^2\\0&\alpha-2&3\\0&1&\alpha\end{vmatrix}$$

$\Rightarrow \alpha^2 - 2\alpha - 3 = 0 \Rightarrow \alpha = 3$ or -1
$|||^{ly}$ $\beta = 3$ or -1
But $\alpha > \beta \Rightarrow \alpha = 3$ and $\beta = -1$.

Paragraph for question nos. 16 & 17

Q.16 (C)

Q.17 (D)

16 to 17

(i) $c_1 \to c_1 - c_2$, $c_2 \to c_2 - c_1$, $c_3 \to c_3 - 2c_1$

$$\begin{vmatrix}2&1&2\\1+\alpha&\alpha&\beta\\4-\beta&3-\beta&\alpha+1\end{vmatrix} = \begin{vmatrix}1&1&2\\1&\alpha&\beta\\1&3-\beta&\alpha+1\end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha-1 & \beta-2 \\ 1 & 2-\beta & \alpha-1 \end{vmatrix}$$

$= (\alpha-1)^2 + (\beta-2)^2 = 0 \Rightarrow \alpha = 1, \ \beta = 2, \gamma = 4$

∴ the cubic equation is

$x^3 - 7x^2 + 14x - 8 = 0$

(ii) $$S = \sum_{r=1}^{100}\left(\left(\frac{\alpha}{\beta}\right)^r + \left(\frac{a}{b}\right)^r\right) = \sum_{r=1}^{100}\left(\frac{1}{2}\right)^r + \left(\frac{-1}{2}\right)^r$$

$$= \sum_{n=1}^{50} 2\left(\frac{1}{2}\right)^{2n} = 2 \cdot \frac{\frac{1}{4}\cdot\left(1-\left(\frac{1}{4}\right)^{50}\right)}{\left(1-\frac{1}{4}\right)}$$

$$= \frac{2}{3}\left(1-\frac{1}{2^{100}}\right)$$ **Ans.**

Comprehension # 2 (Q. No. 18 & 20)

Q.18 (A)

Q.19 (D)

Q.20 (A)

$$(_{A\theta}(\alpha, \beta, \gamma) = \begin{vmatrix} \cos(\alpha+\theta) & \sin(\alpha+\theta) & 1 \\ \cos(\beta+\theta) & \sin(\beta+\theta) & 1 \\ \cos(\gamma+\theta) & \sin(\gamma+\theta) & 1 \end{vmatrix}$$

$A_\theta(\alpha, \beta, \gamma) \Rightarrow \sin(\alpha-\beta)\sin(\beta-\gamma)\sin(\gamma-\alpha) = k$

⇒ which is independent of θ

18 If $a = A_{\pi/2}(\alpha, \beta, \gamma)$ & $b = A_{\pi/3}(\alpha, \beta, \gamma)$
so a = b (Independent of θ)

19 $A_\theta^2 + A_\phi^2 - 2(A_{\theta+\phi})^2 = k^2 + k^2 - 2k^2 = 0$

20 If α, β, γ are fixed then $y = A_x(\alpha, \beta, \gamma)$ = constant
which is a straight line parallel to x-axis.

Q.21 (A) → Q; (B) → R; (C) → R; (D) → P

$$p(\theta) = \begin{vmatrix} -\sqrt{2} & \sin\theta & \cos\theta \\ 1 & \cos\theta & \sin\theta \\ -1 & \sin\theta & -\cos\theta \end{vmatrix} = \sqrt{2} + \sin 2\theta + \cos$$

$$2\theta \Rightarrow \sqrt{2} + \left[-\sqrt{2}, \sqrt{2}\right] = \left[0, 2\sqrt{2}\right]$$

$$q(\theta) = \sqrt{2}\begin{vmatrix} \sin 2\theta & 1 & 1 \\ \cos 2\theta & 2 & 3 \\ \cos 2\theta & 3 & 5 \end{vmatrix} = \sqrt{2}(\sin 2\theta - \cos 2\theta)$$

$$\Rightarrow \sqrt{2}\left[-\sqrt{2}, \sqrt{2}\right] = [-2, 2]$$

$$r(\theta) = \begin{vmatrix} \cos\theta & \sin\theta & \cos\theta \\ -\sin\theta & \cos\theta & \sin\theta \\ -\cos\theta & -\sin\theta & \cos\theta \end{vmatrix} = 2\cos\theta \Rightarrow [-2, 2]$$

$$\text{and } s(\theta) = \begin{vmatrix} \sec^2\theta & 1 & 1 \\ \cos^2\theta & \cos^2\theta & \operatorname{cosec}^2\theta \\ 1 & \cos^2\theta & \cot^2\theta \end{vmatrix} = (\sin^2\theta - 1)^2$$

⇒ [0, 1]

Q.22 (A) → (s); (B) → (p); (C) → (p); (D) → (p)

$$(A)\ \begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$$

$$\Rightarrow [1 + 4x + 3 \ \ 2 + 5x + 2 \ \ 3 + 6x + 5]\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 4+4x & 5x+4 & 6x+8 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$$

$\Rightarrow 4 + 4x + 10x + 8 + 18x + 24 = 0$

$\Rightarrow 32x + 36 = 0$

$\Rightarrow x = -9/8$

(B) $\mu^2 = 49 \quad \Rightarrow \mu^2 = \pm 7$

(C) $(A - \lambda I) = 0 \quad \Rightarrow \lambda^2 - 4\lambda + 5 = 0$

$\therefore A^2 - 4A + 5I = 0$

$$(D)\ \text{Let } a = b = c = 1 \quad \Rightarrow \begin{vmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{vmatrix} = k.27$$

$54 = 27k \quad \Rightarrow k = 2$

Alter : $$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Applying $C_1 \to C_2 - C_3$, $C_2 \to C_2 - C_3$

$$(a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-b-a & c-a-b & (a+b)^2 \end{vmatrix}$$

Applying $R_3 \to R_3 - (R_1 + R_2)$

$$(a + b + c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

$$= \frac{(a+b+c)^2}{ab} \begin{vmatrix} a(b+c-a) & 0 & a^2 \\ 0 & b(c+a-b) & b^2 \\ -2ab & -2ab & 2ab \end{vmatrix}$$

$$2ab\ (a + b + c)^2 \begin{vmatrix} (b+c-a) & 0 & a \\ 0 & (c+a-b) & b \\ -1 & -1 & 1 \end{vmatrix} = 2abc$$

$(a + b + c)^3$

Q.23 (A) → P, Q, T ; (B) → S; (C) → P, R ; (D) → R

Here 24 matrices are possible.

Values of determinants corresponding to these matrices are as follows :

$\begin{vmatrix} 1 & 0 \\ 4 & 2 \end{vmatrix}$ = 2 (4 matrices), $\begin{vmatrix} 1 & 0 \\ 2 & 4 \end{vmatrix}$ = 4 (4 matrices),

$\begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix}$ = 8 (4 matrices)

And 12 more matrices are there, values of whose determinants are –2, –4, –8.

(A) Possible non-negative values of det. (A) are 2, 4, 8.

(B) Sum of these 24 determinants is 0.

(C) Mod. (det(A)) is least $\therefore |A| = \pm 2$

$\Rightarrow |\text{adj (adj (adj } (A))| = |A|^{(n-1)^3} = \pm 2$

(D) Least value of det.(A) is –8

Now, $|4A^{-1}| = 16 \dfrac{1}{|A|} = \dfrac{16}{-8} = -2$

NUMERICAL VALUE BASED

Q.1 [34]

Put $x = -1$

$$A - B + C - D + E = \begin{vmatrix} -2 & -2 & -4 \\ 0 & 3 & -4 \\ 2 & 3 & -3 \end{vmatrix}$$

$= -2(-9 + 12) + 2(8 + 12) = -6 + 40 = 34.$

Q.2 [0]

Let A is the first term and D is the common difference of corresponsding A.P. then

$\dfrac{1}{a} = A + (P - 1)D;$ $\dfrac{1}{b} = A + (q - 1)D;$

$\dfrac{1}{c} = A + (r - 1)D$

$$\text{Let } \Delta = \begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = abc \begin{vmatrix} 1/a & 1/b & 1/c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 - D(R_2) - (A - D)R_3$

$$\Delta = abc \begin{vmatrix} 0 & 0 & 0 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Q.3 [9]

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$f(x) = -(a_2 - x)(a_3 - x) = -x^2 + (a_2 + a_3)x - a_2a_3$

$|a_2 - a_3| = \dfrac{\sqrt{D}}{|a|} = 6 \Rightarrow \sqrt{D} = 6$

Max. $= \dfrac{-D}{4a} = \dfrac{36}{4} = 9.$]

Q.4 [13]

$$\text{Put } \lambda = 0, \text{ we get } E = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix} = 21$$

and

$$A + \frac{B}{\lambda} + \frac{C}{\lambda^2} + \frac{D}{\lambda^3} + \frac{E}{\lambda^4} = \begin{vmatrix} 1+\frac{3}{\lambda} & 1-\frac{1}{\lambda} & 1+\frac{3}{\lambda} \\ 1+\frac{1}{\lambda^2} & 5+\frac{2}{\lambda} & 1-\frac{3}{\lambda} \\ 1-\frac{3}{\lambda^2} & 1+\frac{4}{\lambda} & 3 \end{vmatrix}$$

$$\text{Take } \lambda \to \infty, \text{ we get } A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 8$$

$\therefore (E - A) = 21 - 8 = 13$

Q.5 [50]

Sol. $\mathbf{D} = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$

$\Rightarrow \dfrac{1}{a_1}, \dfrac{1}{a_2}, \ldots\ldots, \dfrac{1}{5}, \dfrac{1}{4}, \ldots\ldots, \dfrac{1}{a_9}$ are in A.P.

$$d = \frac{1}{4} - \frac{1}{5} = \frac{1}{20} \quad \Rightarrow \quad \frac{1}{5} = \frac{1}{a} + \frac{3}{20}$$

$$\Rightarrow \frac{1}{a_1} = \frac{1}{20} \quad \therefore \quad \frac{1}{a_n} = \frac{1}{a_1} + \frac{(n-1)}{20} = \frac{n}{20}$$

$$\Rightarrow \quad a_n = \frac{20}{n}$$

Hence, **D** = $\begin{vmatrix} 20 & \frac{20}{2} & \frac{20}{3} \\ \frac{20}{4} & \frac{20}{5} & \frac{20}{6} \\ \frac{20}{7} & \frac{20}{8} & \frac{20}{9} \end{vmatrix} = \frac{(20)^3}{4 \times 7} \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 1 & \frac{4}{5} & \frac{2}{3} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix}$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \frac{(20)^3}{4 \times 7} \begin{vmatrix} 0 & \frac{-3}{10} & \frac{-1}{3} \\ 0 & \frac{-3}{40} & \frac{-1}{9} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix} = \frac{50}{21}$$

$\Rightarrow$ **21D** = 50

Q.6 **[2]**

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

$$\Rightarrow \frac{1}{abc} \begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$

$$\Rightarrow \frac{abc}{abc} \begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$(ab + bc + ca) \begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$\Rightarrow$ (ab + bc + ca)

$$\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ab+bc+ac) & 0 \\ ac+bc & 0 & -(ab+bc+ca) \end{vmatrix}$$

$= (ab + bc + ca)^3$

As per Question

$$(ab + bc + ca)^3 = 4^3$$

$$(\sqrt{(ab + bc + ac)})^6 = 2^6$$

$$\sqrt{(ab + bc + ac)} = 2$$

Q.7 **[250]**

Given $f(x) = \begin{vmatrix} 5+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 5+\cos^2 x & 4\sin 2x \\ (\sin^2 x) & (\cos^2 x) & 5+4\sin 2x \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$f(x) = \begin{vmatrix} 5 & -5 & 0 \\ 0 & 5 & -5 \\ \sin^2 x & (\cos^2 x) & 5+4\sin 2x \end{vmatrix} = 25$$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \sin^2 x & \cos^2 x & 5+4\sin 2x \end{vmatrix}$$

$f(x) = 150 + 100 \sin 2x$

Thus maximum value of $f(x) = 250$. **Ans**

Q.8 [2]

By operating $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$0 = \begin{vmatrix} p-a & b-q & 0 \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix}$$

$0 = (p - a) \{r(q-b) - b(c-r)\} + a (b - q) (c - r)$

$0 = (p - a) (rq - rb) + a(b - q) (c - r) + b(p - a) (r - c)$

$$0 = \frac{r}{r-c} + \frac{b}{q-b} + \frac{a}{p-a}$$

Dividing by $(p - a) (r - c) (q - b)$

$$\Rightarrow \frac{r}{r-c} + \left(\frac{b}{q-b} + 1\right) + \left(\frac{a}{p-a} + 1\right) = 2$$

Therefore $\frac{r}{r-c} + \frac{q}{q-b} + \frac{p}{p-a} = 2$

Q.9 [0]

$D_1 + D_2 = 0$

$$\begin{vmatrix} x & a & b \\ -1 & 0 & x \\ x & 2 & 1 \end{vmatrix} - \begin{vmatrix} cx^2 & 2a & -b \\ -1 & 0 & x \\ x & 2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-cx^2 & -a & 2b \\ -1 & 0 & x \\ x & 2 & 1 \end{vmatrix} = 0$$

$(x – cx^2) (–2x) + a(–1 – x^2) + 2b(–2) = 0$
$–2x^2 + 2cx^3 – a – ax^2 – 4b = 0$
$2cx^3 – (a + 2)x^2 – (a + 4b) = 0$
The above equation is satisfied by four different values of x,
$\therefore$ It is an identity.

$c = 0, a + 2 = 0 \Rightarrow a = –2, \quad a + 4b = 0 \Rightarrow b = \dfrac{1}{2}$

$\therefore \quad a + 4b + c = 0.$

Q.10 [12]
Each term is zero independently.

$\Rightarrow a = \dfrac{\pi}{4}, b = 1, c = 0$

$\therefore$ Required determinant = $\begin{vmatrix} 0 & 1/2 & 1 \\ 3 & 1 & 1 \\ 0 & 5 & 2 \end{vmatrix} = 12$

KVPY
PREVIOUS YEAR'S

Q.1 (B)
$A^k = I, B^\ell = 0$ (det (B) = 0)
$\Rightarrow$ det (AB) = 0

Q.2 (D)
Determinant =

$$\begin{vmatrix} (2015-1)^{2014} & (2015)^{2015} & (2015+1)^{2016} \\ (2015+2)^{2017} & (2020-2)^{2018} & (2020-1)^{2019} \\ (2020)^{2020} & (2120+1)^{2021} & (2020+2)^{2022} \end{vmatrix}$$

Determinant of remainder =

$$\begin{vmatrix} (1)^{2014} & 0 & 1 \\ 2^{2017} & 2^{2018} & (-1)^{2019} \\ 0 & 1^{2021} & 2^{2022} \end{vmatrix}$$

$=1\{2^{4040} + 1\} + 1\{2^{2017}\}$
$= \{(4)^{2020} + 1\} + 2 . 2^{2016}$
$\Rightarrow (5 – 1)^{2020} + 1 + 2 . 4^{1008}$
$= (5 – 1)^{2020} + 1 + 2. (5 – 1)^{1008}$
remainder, $(–1)^{2020} + 1 + 2 . (–1)^{1008}$
$= 1 + 1 + 2 = 4$

Q.3 (B)

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & x^2 & x^4 \\ 1 & x^3 & x^6 \end{vmatrix} = 3x^4 \Rightarrow (x – x)^2 (x^2 – x^3) (x^3 – x) = 3x^4$$

$\Rightarrow$ x = 0 or $(1 – x)^2 (x^2 – 1) = 3$
$\Rightarrow x^4 – 2x^3 + 2x – 4 = 0$
$\Rightarrow (x – 2) (x^3 + 2) = 0$
$\Rightarrow$ integer values are 0, 2

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)

$$D = \begin{vmatrix} a^2 + 3a + 2 & a+1 & 1 \\ a^2 + 5a + 6 & a+2 & 1 \\ a^2 + 7a + 12 & a+3 & 1 \end{vmatrix}$$

$R_2 \to R_2 – R_1$
$R_3 \to R_3 – R_1$

$$D = \begin{vmatrix} a^2 + 3a + 2 & a+1 & 1 \\ 2a + 4 & 1 & 0 \\ 4a + 10 & 2 & 0 \end{vmatrix} = 4a + 8 – 4a – 10 = –2$$

Q.2 (1)

$$\Delta = \begin{vmatrix} 3 & 2 & -k \\ 1 & -2 & 3 \\ 1 & 2 & -3 \end{vmatrix} = 0 \Rightarrow k = 3$$

$$\Delta_x = \begin{vmatrix} 10 & 2 & -3 \\ 3 & -2 & 3 \\ 5m & 2 & -3 \end{vmatrix} = 0$$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 1 & -2 & 3 \\ 1 & 5m & -3 \end{vmatrix} = 6(7 – 10m)$$

$$\Delta_z = \begin{vmatrix} 3 & 2 & 10 \\ 1 & -2 & 3 \\ 1 & 2 & 5m \end{vmatrix} = 4(7 – 10m)$$

Hence, k = 3 and $m \neq \dfrac{7}{10}$

Q.3 (2)

$$D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$$

=20–2(25)–3(–10)
=20–50+30=0

$$D_1 \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ C & -2 & 7 \end{vmatrix}$$

=20a–2(7b+11c)–3(–2b–6c)
=20a–14b–22c+6b+18c
=20a–8b–4c
=4(5a–2b–c)

$$D_2 \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix}$$

=7b+11c–a(25)–3(2c–b)
=7b+11c–25a–6c+3b
= –25a+10b+5c
= –5(5a–2b–c)

$$D_3 \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix}$$

=6c+2b–2(2c–b)–10a
=–10a+4b+2c
=–2(5a–2b–c)
for infinite solution
$D=D_1=D_2=D_2=0$
$\Rightarrow$ 5a=2b+c

Q.4 (3)
$C_1 + C_2 \to C_1$

$$\begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_1 - R_2 \to R_1$

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

Open w.r.t. R_1
– (2 sin 2x – cos 2x)
cos 2x – 2 sin 2x = f(x)

$$f(x)\big|_{\max} = \sqrt{1+4} = \sqrt{5}$$

Q.5 (1)

$$\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$$

$R_2 \to R_1 + R_3 - 2R_2$

$$\Rightarrow \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 0 & k-6\sqrt{2} & 0 \\ 5 & k & z \end{vmatrix} = 0$$

$\Rightarrow (k - 6\sqrt{2})(3z - 5x) = 0$
if $3z - 5x = 0 \Rightarrow 3(x + 2d) - 5x = 0$
$\Rightarrow x = 3d$ (Not possible)
$\Rightarrow k - 6\sqrt{2}$ $\Rightarrow k^2 = 72$ **Option (1)**

Q.6 (2)

$$\log_{10}(4^x - 2) = 1 + \log_{10}\left(4^x \quad \frac{18}{5}\right)$$

$$(4^x - 2) = 10\left(4 \quad \frac{18}{5}\right)$$

$(4^x)^2 + 4 - 4(4^x) - 32 = 0$
$(4^x - 16)(4^x + 2) = 0$
$4^x = 16$
$x = 2$

$$\begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix} = 3(-2) - 1(0-4) + 4(1)$$

$= -6 + 4 + 4 = 2$

Q.7 (4)

Q.8 (1)

Q.9 [16]

Q.10 (2)

Q.11 [5]

Q.12 [6]

Q.13 [1]

Q.14 (2)

Q.15 (3)

$$\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix}$$

$= -1(a^2 + 10) - 1(-3a - 10) + 2(-6 + 2a)$
$= -a^2 - 10 + 3a + 10 - 12 + 4a$
$\Delta = -a^2 + 7a - 12$
$\Delta = -[a^2 - 7a + 12]$
$\Delta = -[(a - 3)(a - 4)]$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix}$$

$= 0 - 1(-a - 35) + 2(-2 + 7a)$
$\Rightarrow a + 35 - 4 + 14a$
$15a + 31$
Now $\quad \Delta = 15a + 31$
For inconsistent $\Delta = 0 \therefore a = 3, a = 4$
and for a = 3 and 4 $\Delta_1 \neq 0$
$n(S_1) = 2$
For infinite solution : $\Delta = 0$
and $\Delta_1 = \Delta_2 = \Delta_3 = 0$
Not possible
$\therefore n(S_2) = 0$

Q.16 (2)

Q.17 (4)
Q.18 (3)

Q.19 (2)

Q.20 (2)

Q.21 [36]

Q.22 (4)

Q.23 (1)

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (B,C)
$R_3 \to R_3 - R_2, R_2 \to R_2 - R_1$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix} = -648\alpha$$

$R_3 \to R_3 - R_2$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 2 & 2 & 2 \end{vmatrix} = -648\alpha$$

$C_3 \to C_3 - C_2, C_2 \to C_2 - C_1$

$$\begin{vmatrix} (1+\alpha)^2 & \alpha(2+3\alpha) & \alpha(2+5\alpha) \\ 3+2\alpha & 2\alpha & 2\alpha \\ 2 & 0 & 0 \end{vmatrix} = -648\alpha$$

$\Rightarrow \; 2\alpha^2(2+3\alpha) - 2\alpha^2(2+5\alpha) = -324\alpha$
$\Rightarrow \; -4\alpha^3 = -324\alpha$
$\Rightarrow \; \alpha = 0, \pm 9$

Q.2 (B, C)

$$|P| = 12\alpha + 20$$

$$\text{adj } P = \begin{bmatrix} 5\alpha & -10 & -\alpha \\ 3\alpha & -6 & -(3\alpha+4) \\ 10 & 12 & 2 \end{bmatrix}$$

$$\because \frac{Q}{k} = \frac{adj\,P}{|P|} \qquad \Rightarrow \qquad Q = \frac{k}{|P|} adj\ P$$

$$\because q_{23} = -\frac{k}{8} \Rightarrow \frac{(3\alpha+4)k}{(12\alpha+20)} = \frac{k}{8} \Rightarrow \alpha = -1$$

Also $|Q| = \dfrac{k^3}{|P|} \qquad \Rightarrow k = 4$

Hence, (b, c)

Q.3 [2]

$$\begin{vmatrix} x & x^2 & 1 \\ 2x & 4x^2 & 1 \\ 3x & 9x^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ 2x & 4x^2 & 8x^3 \\ 3x & 9x^2 & 27x^3 \end{vmatrix} = 10$$

$$\Rightarrow \; x^3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$$

$\Rightarrow \; 2x^3 + 12x^6 = 10$

$\Rightarrow \; x^3 = \dfrac{5}{6}, -1$

Hence, no. of distinct x = 2

Q.4 [4]

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \underbrace{(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)}_{x} - \underbrace{(a_3b_2c_1 + a_2b_1c_3 + a_1b_3c_2)}_{y}$$

Now if $x \le 3$ and $y \ge -3$
the D can be maximum 6

But it is not possible

as $x = 3 \Rightarrow$ each term of x =1

and y = 3 $\Rightarrow$ each term of y = –1

$$\Rightarrow \prod_{i=1}^{3} a_i b_i c_i = 1 \text{ and} \prod_{i=1}^{3} a_i b_i c_i = -1$$

which is contradicition

so now next possibility is 4

which is obtained as

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 1(1+1) - 1(-1-1) + 1(1-1) = 4$$

Q.5 (A,B,C)

$|I - EF| \neq 0$; $G = (I - EF)^{-1} \Rightarrow G^{-1} = I - EF$

Now, $G.G^{-1} = I = G^{-1} G$

$\Rightarrow$ G (I – EF) = I = (I – EF) G

$\Rightarrow$ G – GEF) = I = G – EFG

$\Rightarrow$ GEF = EFG [G is Correct]

(I – FE) (I + FGE) = I + FGE – FE – FEFGE

= I + FGE – FE – F (G – I) E

= I + FGE – FE – FGE + FE

= I [(B) is Correct]

(So 'D' is Incorrect)

We have

(I – FE) (I + FGE) = I ....(I)

Now

FE(I + FGE)

= FE + FEFGE

= FE + F(G – I)E

= FE + FGE – FE

= FGE

$\Rightarrow$ |FE| |I + FGE| = |FGE|

$$\Rightarrow |FE| \times \frac{1}{|I - FE|} = |FGE| \text{ (from (1))}$$

$\Rightarrow$ |FE| = |I – FE| |FGE|

(option (A) is correct)

Probability

EXERCISES

ELEMENTRY

Q . 1 (3)

Probability of getting 1 in first throw $= \frac{1}{6}$

Probability of not getting 1 in second throws $= \frac{5}{6}$

Both are independent events, so the required probability $= \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$.

Q . 2 (2)

Total number of ways = 36

Favourable numbers of cases are $(1\ 4), (2,\ 3), (3,\ 2),$

$(4,\ 1), (1,\ 5), (2,\ 4), (3,\ 3), (4,\ 2), (5,\ 1) = 9$

Hence the required probability $= \frac{9}{36} = \frac{1}{4}$.

Q . 3 (2)

Favourable cases for one are three *i.e.* 2, 4 and 6 and for other are two *i.e.* 3, 6.

Hence required probability $= \left[\left(\frac{3 \times 2}{36}\right) 2 - \frac{1}{36}\right] = \frac{11}{36}$

{As same way happen when dice changes numbers among themselves}

Q . 4 (4)

Let *R* stand for drawing red ball B for drawing black ball and W for drawing white ball.

Then required probability

$$= P(WWR) + P(BBR) + P(WBR) + P(BWR) + P(WRR) +$$

$$P(BRR) + P(RWR) + P(RBR).$$

$$= \frac{3.2.2}{8.7.6} + \frac{3.2.2}{8.7.6} + \frac{3.3.2}{8.7.6} + \frac{3.3.2}{8.7.6} + \frac{3.2.1}{8.7.6}$$

$$+ \frac{3.2.1}{8.7.6} + \frac{2.3.1}{8.7.6} + \frac{2.3.1}{8.7.6}$$

$$= \frac{2}{56} + \frac{2}{56} + \frac{3}{56} + \frac{3}{56} + \frac{1}{56} + \frac{1}{56} + \frac{1}{56} + \frac{1}{56} = \frac{1}{4}$$

Q . 5 (4)

P (at least 1H) = 1 – P (No head)

= 1 – P (four tail)

$= 1 - \frac{1}{16} = \frac{15}{16}$

Q . 6 (2)

Let 100 students studying in which 60 % girls and 40 % boys.

Boys = 40, Girls = 60

25% of boys offer Maths = $\frac{25}{100} \times 40 = 10$ Boys

10% of girls offer Maths = $\frac{10}{100} \times 60 = 6$ Girls

It means, 16 students offer Maths.

$\therefore$ Required probability $= \frac{6}{16} = \frac{3}{8}$.

Q . 7 (1)

Total number of ways $=2^n$

If head comes odd times, then favourable ways = 2^{n-1}

$\therefore$ Required probability $= \frac{2^{n-1}}{2^n} = \frac{1}{2}$.

Q . 8 (3)

Here $P(A) = \frac{3}{4}, P(B) = \frac{4}{5}$

$\therefore$ Required probability

$= P(A).P(\overline{B}) + P(\overline{A}).P(B) = \frac{7}{20}$.

Q . 9 (3)

Required probability $= \frac{{}^3C_3 + {}^7C_3 + {}^4C_3}{{}^{14}C_3}$

$= \frac{1 + 35 + 4}{14.13.2} = \frac{40}{14.26} = \frac{10}{91}$.

Q . 1 0 (2)

Total ways of arrangements $= \frac{8!}{2!\,.\,4!}$

• w • x • y • z •

Now 'S' can have places at dot's and in places of w, x, y, z we have to put 2A's, one I and one N.

Therefore favourable ways $= 5\left(\frac{4!}{2!}\right)$

Hence required probability $= \frac{5.4!\ 2!\ 4!}{2!\ 8!} = \frac{1}{14}$.

Q.11 (3)

n=Total number of ways $=6^5$

A total of 12 in 5 throw can be obtained in following two ways –

(i) One blank and four $3's = {}^5C_1 = 5$

or (ii) Three $2's$ and two $3's = {}^5C_2 = 10$

Hence, the required probability $= \frac{15}{6^5} = \frac{5}{2592}$.

Q.12 (2)

m rupee coins and n ten paise coins can be placed in a line in $\frac{(m+n)!}{m!\,n!}$ ways.

If the extreme coins are ten paise coins, then the remaining $n-2$ ten paise coins and m one rupee coins can be arragned in a line in $\frac{(m+n-2)!}{m!(n-2)!}$ ways.

Hence the required probability

$$= \frac{\frac{(m+n-2)!}{m!(n-2)!}}{\frac{(m+n)!}{m!\,n!}} = \frac{n(n-1)}{(m+n)(m+n-1)}.$$

Q.13 (1)

Required probability $= \frac{{}^3C_1}{{}^7C_1} \times \frac{{}^2C_1}{{}^6C_1} = \frac{1}{7}$

Q.14 (4)

3 cards are drawn out of 26 red cards (favourable)

$$= \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{26!}{3!\,23!} \times \frac{3!\,49!}{52!} = \frac{2}{17}.$$

Q.15 (3)

Required probability $= \frac{{}^{37}C_2}{{}^{38}C_3} = \frac{\binom{37}{2}}{\binom{38}{3}}$.

Q.16 (4)

Required probability

$$= \frac{{}^5C_1\,{}^4C_1}{{}^{12}C_1\,{}^{12}C_1} + \frac{{}^7C_1\,{}^8C_1}{{}^{12}C_1\,{}^{12}C_1} = \frac{20+56}{144} = \frac{76}{144}$$

Q.17 (3)

Since we have $P(A+B) = P(A)+P(B)-P(AB)$

$\Rightarrow 0.7 = 0.4 + P(B) - 0.2 \Rightarrow P(B) = 0.5$.

Q.18 (4)

Required probability is

$P(\text{Red}+\text{Queen}) - P(\text{Red} \cap \text{Queen})$

$= P(\text{Red}) + P(\text{Queen}) - P(\text{Red} \cap \text{Queen})$

$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$.

Q.19 (1)

Since we have

$P(A \cup B) + P(A \cap B) = P(A) + P(B) = P(A) + \frac{P(A)}{2}$

$\Rightarrow \frac{7}{8} = \frac{3P(A)}{2} \Rightarrow P(A) = \frac{7}{12}$.

Q.20 (3)

$1 - P(A' \cap B') = 0.6,\ \ P(A \cap B) = 0.3,$ then

$P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$

$\Rightarrow 1 - P(A \cap B) = P(A') + P(B') - 0.4$

$\Rightarrow P(A') + P(B') = 0.7 + 0.4 = 1.1$.

Q.21 (2)

$P(A') = 0.3$, $\therefore$ $P(A) = 0.7$

$P(B') = 0.6$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$

$P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$= 0.7 + 0.6 - 0.5 = 0.8$.

Q.22 (1)

$$P\left(\frac{\bar{B}}{\bar{A}}\right) = \frac{1 - P(A \cup B)}{P(\bar{A})} = \frac{1 - \frac{23}{60}}{1 - \frac{1}{3}} = \frac{37}{60} \times \frac{3}{2} = \frac{37}{40}.$$

Q.23 (1)

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{(1/10)}{(1/4)} = \frac{2}{5}.$$

Q.24 (3)

Let $P(A) =$ probability of a boy in two children $= \frac{3}{4}$

Because cases are $BB, BG, GB, GG = 4$

Favourable cases are $BB, BG, GB = 3$

The probability that the second child is also boy is

$P(A \cap B) = \frac{1}{4}$

We have to find $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{3/4} = \frac{1}{3}$.

Q.25 (2)

We define the following events :

A_1 : Selecting a pair of consecutive letter from the word LONDON.

A_2 : Selecting a pair of consecutive letters from the word CLIFTON.

E : Selecting a pair of letters 'ON'.

Then $P(A_1 \cap E) = \frac{2}{5}$; as there are 5 pairs of consecutive letters out of which 2 are ON.

$P(A_2 \cap E) = \frac{1}{6}$; as there are 6 pairs of consecutive letters of which one is ON.

$\therefore$ The required probability is

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E) + P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{6}} = \frac{12}{17}.$$

Q.26 (2)

$$P(A) = \frac{40}{100}, \quad P(B) = \frac{25}{100} \quad \text{and} \quad P(A \cap B) = \frac{15}{100}$$

So

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{15/100}{40/100} = \frac{3}{8}.$$

Q.27 (1)

$$P(A \cap B) = P(A).P(B) = \frac{1}{6}$$

$$P(\bar{A} \cap \bar{B}) = \frac{1}{3} = 1 - P(A \cup B)$$

$$\Rightarrow \frac{1}{3} = 1 - [P(A) + P(B)] + \frac{1}{6} \Rightarrow P(A) + P(B) = \frac{5}{6}.$$

Hence P(A) and P(B) are $\frac{1}{2}$ and $\frac{1}{3}$.

Q.28 (1)

(i) This question can also be solved by one student

(ii) This question can be solved by two students simultaneously

(ii) This question can be solved by three students all together.

$$P(A) = \frac{1}{2},\ P(B) = \frac{1}{4}, P(C) = \frac{1}{6}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$-[P(A).P(B) + P(B).P(C) + P(C).P(A)] + [P(A).P(B).P(C)]$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{6} - \left[\frac{1}{2} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{2}\right] + \left[\frac{1}{2} \times \frac{1}{4} \times \frac{1}{6}\right] = \frac{33}{48}$$

Q.29 (1)

Let E be the event in which all three coins shows tail and F be the event in which a coin shows tail.

$\therefore$ $F = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$

and $E = \{TTT\}$ Required probability

$$= P(E/F) = \frac{P(E \cap F)}{P(E)} = \frac{1}{7}.$$

Q.30 (4)

$$P\left(\frac{B}{A}\right) = \frac{1}{2} \Rightarrow \frac{P(B \cap A)}{P(A)} = \frac{1}{2} \Rightarrow P(B \cap A) = \frac{1}{8}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \Rightarrow P(B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{8} = P(A).P(B)$$

$\therefore$ Events *A* and *B* are independent.

$$\text{Now, } P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = \frac{P(A')P(B)}{P(B)} = \frac{3}{4}$$

$$\text{and } P\left(\frac{B'}{A'}\right) = \frac{P(B' \cap A')}{P(A')} = \frac{P(B')P(A')}{P(A')} = \frac{1}{2}.$$

Q.31 (4)

$$\text{Let } P(\text{fresh egg}) = \frac{90}{100} = \frac{9}{10} = p$$

$$P(\text{rotten egg}) = \frac{10}{100} = \frac{1}{10} = q\ ; \quad n = 5, \quad r = 5$$

So the probability that none egg is rotten

$$= {}^5C_5 \left(\frac{9}{10}\right)^5 . \left(\frac{1}{10}\right)^0 = \left(\frac{9}{10}\right)^5 .$$

Q.32 (1)

Required probability

$$= {}^8C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^7 + {}^8C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^8 = \frac{27}{20}\left(\frac{19}{20}\right)^7$$

Q.33 (4)

We have $p = \frac{3}{4} \Rightarrow q = \frac{1}{4}$ and $n = 5$

Therefore required probability

$$= {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2 + {}^5C_4 \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right) + {}^5C_5 \left(\frac{3}{4}\right)^5$$

$$= \frac{10.27}{4^5} + \frac{5.81}{4^5} + \frac{243}{4^5} = \frac{270 + 405 + 243}{1024} = \frac{459}{512}.$$

Q.34 (1)

The probability that student is not swimmer

$p = \frac{1}{5}$ and probability that student is swimmer $q = \frac{4}{5}$

$\therefore$ Probability that out of 5 students 4 are swimmer

$$= {}^5C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^{5-4} = {}^5C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right).$$

Q.35 (3)

Probability of failure $= \frac{1}{3}$

Probability for getting success $= \frac{2}{3}$

$\therefore$ Required probability

$$= {}^4C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0 + {}^4C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)$$

$$= \left(\frac{2}{3}\right)^4 + 4\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right) = \frac{16}{27}$$

Q.36 (4)

We are given that $n = 3, \quad p = \frac{1}{6},$

$$q = \frac{5}{6}$$

Mean $= np = 3 \times \frac{1}{6} = \frac{1}{2}$

Variance $= nqp = 3 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{12}.$

Q.37 (2)

Obviously, $p = \frac{2}{6} = \frac{1}{3} \Rightarrow q = 1 - \frac{1}{3} = \frac{2}{3},$

also $n = 2.$ Therefore, variance

$$= npq = 2 \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{9}.$$

Q.38 (1)

For binomial distribution, mean $=$ np and variance $=$ npq

$n = 3 \;\; p = \frac{2}{6} = \frac{1}{3}, \;\; q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

So, mean $(\mu) = 3 \times \frac{1}{3} = 1$

Variance $(\sigma^2) = 3 \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{3}$

Q.39 (1)

Let E denote the event that a six occurs and A the event that the man reports that it is a '6', we have

$P(E) = \frac{1}{6}, \; P(E') = \frac{5}{6}, \; P(A/E) = \frac{3}{4}$ and

$P(A/E') = \frac{1}{4}$

From Baye's theorem,

$$P(E/A) = \frac{P(E).P(A/E)}{P(E).P(A/E) + P(E').P(A/E')}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}.$$

Q.40 (3)

Let *A* be the event of selecting bag *X*, *B* be the event of selecting bag *Y* and *E* be the event of drawing a white ball, then $P(A) = 1/2, P(B) = 1/2$,

$P(E/A) = 2/5 \;\; P(E/B) = 4/6 = 2/3$.

$$P(E) = P(A)P(E/A) + P(B)P(E/B) = \frac{1}{2}\cdot\frac{2}{5} + \frac{1}{2}\cdot\frac{2}{3} = \frac{8}{15}.$$

JEE-MAIN

OBJECTIVE QUESTIONS

Q.1 (3)

Since sum of 1+2+3+......9 $= \frac{9 \times 10}{2} = 45$ is divisible by 9, hance all no. will be divisible by 9.

Q.2 (2)

$$x + \frac{100}{x} > 50$$

$$\left.\begin{array}{l} x = 1\text{Satisfies} \\ \;\;= 2 \quad '' \end{array}\right]2\text{ Numbers.} \quad \text{Ans } P = \frac{55}{100}$$

= 3 does'nt satisfies

$$\left.\begin{array}{l} = 48 \;\; \text{Satisfies} \\ = 100\text{Satisfies} \end{array}\right]53\text{ Numbers}$$

Q.3 (1)

Max sum $= 12$

$$\left.\begin{array}{l} 6 + 6 = 12 \\ 6 + 5 = 11 \\ 6 + 4 = 10 \\ 5 + 5 = 10 \\ 5 + 6 = 11 \\ 4 + 6 = 10 \end{array}\right\} \; 6 \text{ cases}$$

$$P = \frac{6}{36} = \frac{1}{6} = \frac{1}{6}$$

Q.4 (1)

$$\frac{\dfrac{2n-2!}{n-1!\,n-1!2!}\times 2!}{\dfrac{2n!}{n!\,n!2!}} = P$$

Q.5 (1)

Given that,

$\alpha^2 + \beta^2 = \alpha + \beta$ & $\alpha^2\,\beta^2 = \alpha\,\beta$

4 possibilities

$(1, 1), (1, 0), (0, 0), (\omega, \omega^2)$

Required probability = $\frac{2}{4} = \frac{1}{2}$

Q.6 (4)

Total ways in which 5 persons can exit at 8 floor = 8^5

(each has 8 options)

No. of ways of selecting 5 floor out of 8 = ${}^8C_5$

No. of ways of exifing at 5 selected floor = 5!

$\therefore$ Probability = $\frac{{}^8C_5 \times 5!}{8^5}$

Q.7 (1)

Required probability

$$= \frac{{}^5C_4 \times {}^3C_2 \times {}^2C_1}{{}^{10}C_7} = \frac{1}{4}$$

Q.8 **(1)**

${}^4C_1 \times {}^{13}C_9 \times {}^{39}C_4$ = Formula card

site any 9 cards any 4 cards from 39 cards

${}^{52}C_{13}$ = total case

$$P = \frac{{}^4C_1 \times {}^{13}C_9 \times {}^{39}C_4}{{}^{52}C_{13}}$$

Q.9 (4)

P (A) = $\frac{13}{52} = \frac{1}{4}$ Let P(A) is prob of card drawn is spade & P(B) is card drawn is an ace then

P (B) = $\frac{4}{52} = \frac{1}{13}$

Q.10 (4)

Let D = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Each element can be either '0' or '1' Total determinants $= 2 \times 2 \times 2 \times 2 = 16$

Non-Negative Determinants :

$\begin{bmatrix} 1 & b \\ c & 1 \end{bmatrix} \Rightarrow 2 \times 2 = 4$

$\begin{bmatrix} 1 & b \\ c & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & b \\ c & 1 \end{bmatrix} = {}^2C_1 + {}^2C_1 = 4$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Only one outof a,b,c or d is equal to '1'

Q.11 (3)

Throughing both cubes is an independent event and occurring of red 8 blue color on top face is mutually exclusive let there be 'x' blue faces on the second cube.

Given,

$$\underbrace{\frac{1}{6}\times\frac{x}{6}}_{\text{Blue at the top}} + \underbrace{\frac{5}{6}\times\frac{(6-x)}{6}}_{\text{Red at the top}} = \frac{1}{2}$$

$\Rightarrow x + 30 - 5x = 18$

$\Rightarrow 4x = 12 \Rightarrow x = 3$

$\therefore$ No. of red faces $= (6 - x) = (6 - 3) = 3$

Q.12 (4)

$\Rightarrow \frac{1}{6}\times\frac{2}{3} = \frac{1}{9}$

$\Rightarrow \frac{1}{3}\times\left(\frac{1}{6}\times 3\right) = \frac{1}{6}$

$P = \frac{1}{9} + \frac{1}{6} = \frac{2+3}{18} = \frac{5}{18}$

$P_1 = \frac{13}{18}$

adds against = $\frac{13}{5}$

Q.13 (4)

$p_1 + p_2 + p_3 + p_4 = 1$ in in D obvious solution follows

Q.14 (3)

$$\left.\begin{array}{l} 2+6=8 \\ 3+5=8 \\ 4+4=8 \\ 6+2=8 \\ 5+3=8 \end{array}\right] \text{Total} = 5$$

$P = \frac{\text{fav}}{\text{Total}} = \frac{1}{5}$

Q.15 (1)

Required Probability

$$= \frac{3}{9} \times \frac{6}{8} + \frac{6}{9} \times \frac{5}{8} = \frac{6}{9 \times 8}(5+3) = \frac{2}{3}$$

Q.16 (1)

Total = m + n

$$(w, w) \Rightarrow \frac{m}{m+n} \times \frac{m-1}{m+n-1} \qquad ....(1)$$

$$(B, w) \Rightarrow \frac{n}{m+n} \times \frac{m}{m+n-1} \qquad ....(2)$$

Total proof = (w, w) + (B, w)

$$= \frac{m}{m+n(m+n-1)}\ (m+n-1) = \frac{m}{m+n}$$

Q.17 (4)

Required Probability

$$= \frac{{}^{7}C_3 - 2}{{}^{10}C_3} = \frac{11}{40}$$

Q.18 (1)

Required probability

$$= \frac{{}^{3}C_2 \times 1 \times 1}{2^3 - 1} = \frac{3}{7}$$

Q.19 (2)

Required Probability

$$= \frac{1}{2}\left(\frac{{}^{4}C_1 \times 2 + {}^{24}C_1 \times 3 + {}^{36}C_1 \times 4}{32 \times 63}\right)$$

$$= \frac{1}{2}\left(\frac{8 + 72 + 36 \times 4}{32 \times 63}\right) = \frac{1}{2}\left(\frac{1}{9}\right) = \frac{1}{18}$$

Q.20 (4)

$\boxed{A}$

If A ≠ B

1 5

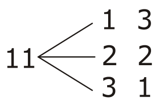
5 1

4 2 22 – 11

$$\frac{4}{36} + \frac{4}{36 \times 36} = \frac{148}{1296}$$

$\boxed{B}$

If A = B



2 4

Q.21 (1)

squared of a no. can have 1, 4, 6, 9, 5

So P = (9/25)

Q.22 (2)

Required probability

$$= \frac{{}^{3}C_1{}^{3}C_1{}^{3}C_1 + {}^{2}C_1{}^{2}C_1{}^{2}C_1 + {}^{3}C_1{}^{2}C_1{}^{2}C_1 \times 2}{5 \times 5 \times 5}$$

$$= \frac{59}{125}$$

Q.23 (1)

Foce cards = 12

Tens = 4

Total removed cards = 16

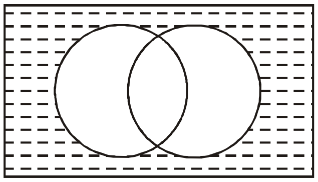
Remaining cards = 36

$\therefore$ P(A) = $\frac{4}{36}$; P(H) = $\frac{9}{36}$; P(S) = $\frac{9}{36}$

$\Rightarrow$ 9P(A) = 4P(H)

Q.24 (3)

$A^c - B$



$= (A \cup B)^C$

Q.25 (1)

. p (A) = $\frac{1}{6}$, p (B) = $\frac{2}{6}$

A ≡ {1, 3, 5 } B≡ { 3, 6 }

B $\not\subset$ A .

B – A = {6} as follows

Q.26 (1)

$$\underbrace{\_\ \_\ \_}_{\text{Letters}}\ \underbrace{\_\ \_\ \_}_{\text{Digits}}$$

$$P(A) = P(\text{letter pallindrome}) = \frac{26 \times 26 \times 1}{26 \times 26 \times 26} = \frac{1}{26}$$

$$P(B) = P(\text{Digit Pallendrome}) = \frac{10 \times 10 \times 1}{10 \times 10 \times 10} = \frac{1}{10}$$

P(Both letter & Dight Pallindrome)

$$P(A\cap B)=\frac{26\times 26\times 1\times 10\times 10\times 1}{26\times 26\times 26\times 10\times 10\times 10}=\frac{1}{260}$$

$\Rightarrow\ P(A\cup B)=P(A)+P(B)=P(A\cap B)$

$$=\frac{1}{10}+\frac{1}{26}-\frac{1}{260}=\frac{7}{52}$$

Q.27 (4)

Let the probabilities of $A\cap B$, A, B & $A\cup B$ be $a-3d$, $a-d$, $a+d$, $a+3d$ in AP

Given, $(a-d)=2d$

$a=3d$

$2P(A)=2(a-d)=2(2d)=4d$

& $P(B)=a+d=3d+d=4d$

$\Rightarrow\ 2P(A)=P(B)$

Q.28 (1)

P(atleast / W) = P(1W, 1M) + P(2W$_1$ OM)

$$=\frac{5\times 8}{^{13}C_2}+\frac{^{5}C_2}{^{13}C_2}=\frac{25}{39}$$

Q.29 (1)

A = (1, 3, 5), A ∩ B = (3, 5)

P (B/A) = P (B ∩ A)/ p(A) = 2/3

Q.30 (4)

P(A) = 1/3 P(B) = 1/4 & P(B) = 3/4

P(T T) = 1/6 1/3 1/4 P(FF) = 5/6 2/3 3/4 $(1/5)^2$

$$P(\text{correct})=\frac{P(TT)}{P(TT)+P(FF)}$$

Q.31 (3)

$P(A\text{ Late})=\frac{1}{5}$ $P(B\text{ Late})=\frac{7}{25}$

P (B is late given that A is late) = $\frac{9}{10}$

(i) neither bus is late

$$P(\overline{A}\cap\overline{B})=P(\overline{A}\cup\overline{B})=1-P(A\cup B)$$

$$\frac{P(B\cap A)}{P(A)}=\frac{9}{10}$$

$$P(B\cap A)=\frac{9}{10}\times P(A)$$

$$=\frac{9}{10}\times\frac{1}{5}=\frac{9}{50}$$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$=\frac{1}{5}+\frac{7}{25}-\frac{9}{50}=\frac{3}{10}$$

$1-P(A\cup B)=7/10$

$$\text{(ii) } P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{9}{50}}{7/15}=\frac{9}{14}$$

Q.32 (2)

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore\ P(A\cap B)=P(A)\cdot P(B)$ because A & B are independent events

$\Rightarrow\ P(A\cup B)=1-P(\overline{A})\cdot P(\overline{B})$

$\Rightarrow\ 0.8=1-(0.7)\cdot(a)$

$\Rightarrow\ (0.7)a=1-0.8=0.2$

$$\Rightarrow\ a=\frac{0.2}{0.7}\qquad\Rightarrow a=\frac{2}{7}$$

Q.33 (3)

1 – P(BB)

1 – 1/2 × 1/2 = 1 – 1/4 = 3/4

Q.34 (2)

$5\to 3$

$$\frac{20}{3}\to\frac{3}{5}\times\frac{20}{3}=4$$

$$P(B)=\frac{4}{10}$$

$5\to 2$

$$\frac{20}{3}\to\frac{2}{5}\times\frac{20}{3}=\frac{8}{3}$$

$$P(C)=\frac{8}{3\times 10}=\frac{4}{15}$$

Q.35 (3)

$$p(A)=\frac{1\times 6}{36}=\frac{1}{36},\qquad p(B)=\frac{6}{36}=\frac{1}{6}$$

$$\left.\begin{matrix}6+1\\5+2\\4+3\end{matrix}\right\}\ A\cap B=\frac{1}{36}\ ;\ p(A\cap B)=p(A)\times p(B)$$

Q.36 (2)

A and B are independent events

$$\therefore\ P(A)=P(B)=\frac{1}{13}$$

Q.37 (1)

$^{3}C_2P^2\ (1–P)=12\ {}^{3}C_3P^3$

$$1-P=4P\Rightarrow\frac{1}{5}=p=\frac{11}{243}$$

Q.38 (3)

2W & 4B

$$P = {}^5C_4 \times \left(\frac{2}{6}\right)^4 \left(\frac{4}{6}\right)^1 + {}^5C_5 \left(\frac{2}{6}\right)^5 = \frac{11}{243}$$

Q.39 (3)

$$\frac{3}{5} \times \frac{2}{4} \times 30 + \frac{3}{5} \times \frac{2}{4} \times 40 + \frac{2}{5}\ \frac{1}{4} \times 20$$

Q.40 (2)

Sol. $$E_A\left[\frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + ...\right] \times 99$$

$$= \frac{99}{6}\ \frac{B6}{1-\frac{25}{36}} = 54$$

Q.41 (3)

A = event it is chosen from A

P(A) = 3/5

P (B) = 2/5

$$P(D) = P(A)\ .\ P\left(\frac{D}{A}\right) + P(B)\ P\left(\frac{D}{B}\right)$$

$$= \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{1}{5}$$

$$= \frac{6}{25} + \frac{2}{25} = \frac{8}{25}$$

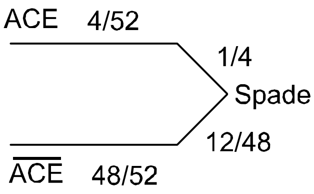
Q.42 (3)

Required Probability

$$= \frac{\left(\frac{1}{4} \times 1\right)}{\left(\frac{3}{4} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{4} \times 1 \times 1 \times 1\right)}$$

$$= \frac{\left(\frac{1}{4}\right)}{\frac{1}{4}\left(\frac{3}{216} + 1\right)} = \frac{216}{219}$$

Q.43 (1)



$$\text{Required probability} = \frac{\frac{4}{52} \times \frac{1}{4}}{\frac{4}{52} \times \frac{1}{4} + \frac{48}{52} \times \frac{12}{48}} = \frac{1}{13}$$

Q.44 (2)

$$\frac{np}{npq} = \frac{3}{2} \Rightarrow q = \frac{2}{3} \Rightarrow p = \frac{1}{3}$$

$$r \le \frac{11}{1+\frac{2}{3}} \qquad \Rightarrow r \le \frac{10}{3}$$

$\Rightarrow r \le 3.33$

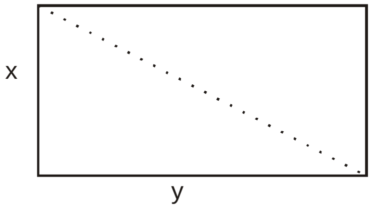
thus 3 succes is most parallal.

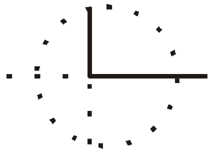
Q.45 (3)

$0 < x < 10\ x^2 + y^2 < 100$

$0 < y < 10$

$$p = \frac{\frac{1}{4}\prod \times 10^2}{10 \times 10}$$





$$\mathbf{P} = \frac{\pi}{4}$$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (A)

Since line are more ${}^NC_M$ are those lies where telegrams will go

${}^NC_M \times M\,! =$ far

Total $= N^M$ [As first telegram can go in any one of n lies]

[As 2nd telegram can go in any one of n lies]

$$P = \frac{{}^NC_M\,M!}{N^M}$$

Q.2 (B)

Total cases $x^8 : (x^0 + x^1 + ...x^6)^4 = \left(\frac{1-x^7}{1-x}\right)^4$

$= (1 - x^7)^4 (1 - x)^{-4}$

$= (1-2x^7)^2 (1-x)^{-4} = (1-4x^7)(1-x)^{-4}$

Total ways $= 7^4\ {}^{4+8-1}C_8 - 4\ {}^{4+1-1}C_1 = {}^{11}C_8 - 4 \times 4$

$= 165 - 16 = 149.$

$$P = \frac{149}{7^4}$$

Q.3 (B)

Unit digit in number	Unit digit in number	Unit digit in product
Odd	Odd	Odd
Odd	Even	Even
Even	Odd	Even
Even	Even	Even

$$p = \frac{3}{4}$$

$$q = \frac{1}{4}$$

$$\frac{p}{q} = 3$$

Q.4 (D)

1^{st} coupeon can be selected in 9 ways

2^{nd} coupeon can be selected in 9 ways

3^{rd} coupeon can be selected in 9 ways

9^7 ways – when 9 is not take

for f= $9^7 - 8^7$

Total = 15^7.

Q.5 (B)

$$\frac{{}^6C_1 \times {}^2C_1}{{}^6C_1 \times {}^6C_1} = \frac{2}{6} = \frac{1}{3}$$

Q.6 (B)

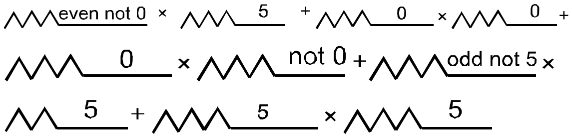
$$\frac{{}^7C_3 \times 1 \times {}^4C_2 \times 1}{7^7} = \frac{30}{7^6}$$

Q.7 (C)

mutually Exclusive event

$$P(A \cup B \cup C) = \frac{1}{6} + \frac{1}{10} + \frac{1}{8} = \frac{47}{120}$$

Q.8 (C)



$$P = \frac{4}{10} \times \frac{1}{10} \times 2 + \frac{1}{10} \times \frac{1}{10} + \frac{1}{10} \times \frac{9}{10} \times 2 + \frac{4}{10} \times \frac{1}{10} \times 2$$

$$+ \frac{1}{10} \times \frac{1}{10} = \frac{9}{25}$$

Q.9 (D)

Light changes in 63 second = 3 times

Light changes in 1 second = 3/63

probability changing light is 3 second

$$= \frac{9}{63} = \frac{1}{7}$$

Q.10 (D)

Required probability

$$= \frac{{}^5C_2}{{}^6C_2} + \frac{{}^5C_1}{{}^6C_2} \times \frac{{}^7C_1}{{}^8C_2} \qquad = \frac{9}{12} = \frac{3}{4}$$

Q.11 (C)

$p(A \cup B) = p(A) + pB) - p(A \cap B)$

$$\frac{5}{6} = \frac{1}{2} + p(B)\ \frac{-1}{2}$$

$$p(B) = \frac{2}{3}$$

$$p(A \cap B) = \frac{1}{3} = p(A)\, p(B)$$

⇒ A & B one independnet

Q.12 (A)

$$n(s) = \frac{20!}{10!\,10!\,2!} \quad P(E) = \frac{n(E)}{n(s)}$$

$$n(E) = \frac{18!}{10!\,8!}$$

Q.13 (C)

$$p(I^{st}\text{ class}) = \frac{2}{3} \times 0.28 + \frac{1}{3} \times 0.25 = 0.27$$

Q.14 (B)

$$\frac{2}{3}\,\frac{{}^3C_1 \times {}^3C_1}{{}^6C_2} + \frac{1 \times 1}{3\ {}^6C_6} = \frac{11}{15}$$

Q.15 (D)

$$p\,(A/B) = \frac{p(A\cap B)}{p(B)} = \frac{0.1+0.1}{0.3} = \frac{2}{3}.$$

similarly evaluate others

Q.16 (C)

Required probability

$$= \frac{7\times 0.2}{7\times 0.2+3\times 0.9} = \frac{14}{41} \simeq 0.34$$

Q.17 (A)

$U1 - 1W + 1B$ $U2 \to 2W + 3B$

$U3 \to 3W + 5B$ $U4 \to 4W + 7B$

$$P(W) = \sum_{i=1}^{4}(u_1)\ P(w/u_i) = \sum_{i=1}^{4}\frac{i^2+1}{34}\ P(w/v_i)$$

$$= \frac{1^2+1}{34}\times\frac{1}{2}+\frac{2^2+1}{34}\times\frac{2}{5}+\frac{3^2+1}{34}\times\frac{3}{8}+\frac{4^2+1}{34}\times\frac{4}{11}$$

$$= \frac{569}{1496}$$

Q.18 (A)

A : 1 ball is W & 3 black balls

B_1 : Urn 1 is chosen

B_2 : Urn 2 is chosen

$$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1)+P(A/B_2)P(B_2)}$$

$$P(B_1/A) = \frac{\frac{1}{2}\times\frac{1}{9}\times\left(\frac{5}{9}\right)^3\times{}^4C_3}{\frac{1}{2}\times\frac{1}{9}\times\left(\frac{5}{9}\right)^3\times{}^4C_3+\frac{1}{2}\times\frac{3}{9}\times\left(\frac{6}{9}\right)^3\times{}^4C_3}$$

$$= \frac{125}{287}$$

Q.19 (C)

KRISHNAGIRI or DHARMAPURI

A = R1 is visible

B_1 = its from KRISHNAGIRI

B_2 = its from DHARMAPURI

$$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1)+P(A/B_2)P(B_2)}$$

$$= \frac{\frac{1}{2}\times\frac{2}{10}}{\frac{1}{2}\times\frac{2}{10}+\frac{1}{2}\times\frac{1}{9}} = \frac{9}{14}$$

Q.20 (A)

due to equal probability theorem $= \frac{m}{m+n}$

Q.21 (C)

a. The probability of S_1 to be among the eight winners is equal to the probability of S_1 winning in the group, which is given by 1/2.

b. If S_1 and S_2 are in the same pair-then exactly one wins. If S_1 and S_2 are in two separate pairs, then for exactly one of S_1 and S_2 to be among the eight winners, S_1 wins and S_2 loses or S_1 loses and S_2 wins.

Now the probability of S_1, S_2 being in the same pair and one wins is (Probability of S_1, S_2 being in the same pair) × (Probability of any one winning in the pair). And the probability of S_1, S_2, being the same pair is

$$\frac{n(E)}{n(S)}$$

The number of ways 16 players are divided into 8 pairs is

$$n(s) = \frac{16!}{(2!)^8\times 8!}$$

The number of ways in which 16 persons can be divided in 8 pairs so that S_1 and S_2 are in same pair is

$$n(E) = \frac{14!}{(2!)^7\times 7!}$$

Therefore, the probability of S_1 and S_2 being in the same pair is

$$\frac{\frac{14!}{(2!)^7\times 7!}}{\frac{16}{(2!)^8\times 8!}} = \frac{2!\times 8}{16\times 15} = \frac{1}{15}$$

The probability any one winning in the pair of S_1, S_2, is P(certain event)=1.

Hence, the probability that the pair of S_1, S_2, being in two pairs separately and any one of S_1, S_2 wins is given by the probability of S_1, S_2 being in two pairs separately and S_1 wins, S_2 loses + the probability of S_1, S_2 wins. It is given by

$$\left[1-\frac{1}{15}\right]\times\frac{1}{2}\times\frac{1}{2}+\left[1-\frac{1}{15}\right]\times\frac{1}{2}\times = \frac{1}{2}\frac{1}{2}\times\frac{14}{15} = \frac{7}{15}$$

Therefore, therequired probabilityis (1/15) + (7/15) + (8/15)

Q.22 (D)

	3	9	7	1	
3					
9					
7				x	
1			x		

Probability $= \dfrac{2}{4\times 4} = \dfrac{2}{16}$

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MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A, B, C, D)

$P(T\,1) = p$

$P(T\,2) = q$

$P(T\,3) = 1/2$

$$\frac{1}{2} = P(T\,1, T\,2) + P(T\,1, T3) + P(T\,1\,T\,2\,T\,3)$$

$$\frac{1}{2} = pq\ 1/2 + p(1-q)\ \frac{1}{2} + pq\ \frac{1}{2}$$

$$\frac{1}{2} = \frac{pq}{2} + p \Rightarrow 1 = pq + 2p.$$

Now, check options.

Q.2 (A, C)

$A = \{1,3,5\}$; $B = \{2,4,6\}$;

$C = \{4,5,6\}$; $D = \{1,2\}$

Q.3 (C, D)

1 1

$P(E) = \dfrac{12}{36}$

2 2

$= \dfrac{1}{3}$

3 3

$P(F) = \dfrac{1}{3}$

4 4

5 5

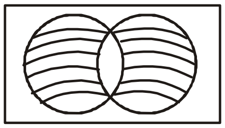
6 6

$P(E\cap F) = \dfrac{2}{36} = \dfrac{1}{18}$

So neither independent
matually Exclusive

Q.4 (A,B,C)

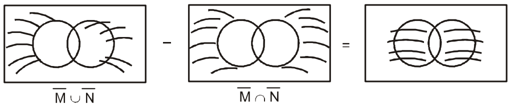
Q.5 (A,C,D)



$P = P(M\cup N) - P(M\cap N)$

$= P(M) + P(N) - 2P(M\cap N)$

(c) $P(\overline{M}\cup\overline{N}) - P(\overline{M}\cup\overline{N})$



Q.6 (C, D)

A & B are independent

$P(A\cup B)^c = 1 - P(A\cup B) = 1 - P(A) - P(B) + P(A\cap B)$

$= P(\overline{A}) - P(B) + P(A)\ P(B) = P(\overline{A}) - P(\overline{A})P(B)$

$= P(\overline{A})P(\overline{B})$

Q.7 (A,B,C,D)

$$P(E_o) = \frac{3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right)}{3!} = \frac{1}{3}$$

$$P(E_1) = \frac{{}^3C_1(1).\ 2!\left(1-\frac{1}{1!}+\frac{1}{2!}\right)}{3!} = \frac{1}{2}$$

$$P(E_2) = \frac{{}^3C_2(1)^2\ .\ 1!\left(1-\frac{1}{1!}\right)}{3!} = 0$$

$$P(E_3) = \frac{{}^3C_3(1)^3}{3!} = \frac{1}{6}$$

So $P(E_0) + P(E_3) = P(E_1)$

$P(E_0)\ .\ P(E_1) = P(E_3)$

$\because$ $E_0 \cap E_1 = \phi$

So $P(E_0\cap E_1) = 0$

So $P(E_0\cap E_1) = P(E_2)$

Q.8 (C, D)

$P(A\cap B) = P(A)\cdot P(B)$

means a and
B are independent
Event

So $P\,P(\overline{A\cup B}) = P(\overline{A}\cap\overline{B})$

$= P(\overline{A})\cdot P(\overline{B})$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = P(A)$$

Q.9 (A,B,C)

Q.10 (A,B,C,D)

(A) $P\left(A/\overline{B}\right) = \frac{P(A \cap \overline{B})}{P\left(\overline{B}\right)}$

$$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

(B) $P(A) + (B) = P(A \cup B) + P(A \cap B)$

$P(A) + P(B) - P(A \cup B) = P(A \cap B)$

So $P(A \cup B) \neq 1$

$P(A) + B(B) - 1 \leq P(A \cap B)$

(C) $P(A) > P(A/B)$

$$P(A) > \frac{P(A \cap B)}{P(B)}$$

$P(A) \cdot P(B) > P(A \cap B)$

Then

$P(A/\overline{B}) > P(A)$

$$\frac{P(A \cap \overline{B})}{P(\overline{B})} > P(A)$$

$$\frac{P(A) - P(A \cap B)}{1 - P(B)} > P(A)$$

$P(A) - P(A \cap B) > P(A) - P(A)\,P(B)$

$P(A \cap B) < P(A)\,P(B)$

product

(D) $\frac{P(A \cap \overline{B})}{P(\overline{B})} + \frac{P(\overline{A} + \overline{B})}{P(\overline{B})}$

$$\frac{P(A) - P(A \cap B) + \overline{P(A \cup B)}}{P(\overline{B})}$$

Q.11 (A,C,D)

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_2 \cap E_1)}{P(E_1)} = P\left(\frac{E_1}{E_2}\right) = \frac{P(E_2 \cap E_1)}{P(E_2)}$$

$$\frac{1}{2} = \frac{P(E_2 \cap E_1)}{1/4}$$

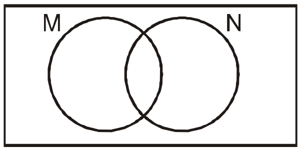
$P(E_2 \cap E_1) = 1/8$

$$P(E_1/E_2) = \frac{1/8}{P(E_2)}$$

$$\frac{1}{4} \quad PE_2 = \frac{1}{2}$$

Q.12 (B, C, D)

A is wrong B is right



C is right

$$P\left(\frac{M}{N}\right) + P\overline{\left(\frac{M}{N}\right)} = 1$$

Q.13 (A,B)

$P(A) = \frac{1}{3}$ Area of region

$P(B) = \frac{1}{3}\, S_1, S_2, S_3$

$P(A \cap B) = \frac{1}{3}$ is equal

Thetigase Shows that A & B are exhaustive Event

Q.14 (A, D)

$$\left.\begin{array}{l} p(x=4) = {}^{n}c_4\left(\frac{1}{2}\right)^n \\ p(x=5) = {}^{n}c_5\left(\frac{1}{2}\right)^n \\ p(x=6) = {}^{n}c_6\left(\frac{1}{2}\right)^n \end{array}\right]$$

$2\,{}^{n}c_5 = {}^{n}c_4 + {}^{n}c_6$

$4\,{}^{n}c_5 = {}^{n+1}c_5 + {}^{n+1}c_6$

$4\,{}^{n}c_5 = {}^{n+2}c_6 + {}^{n}c_6$

$$4 \cdot \frac{n!}{5!\,(n-5)!} = \frac{(n+2)!}{6!(n-4)!}$$

$$4 = \frac{(n+2)(n+1)}{6(n-4)} \Rightarrow 24\,(n\text{-}4) = (n+2)\,(b+1)$$

$n = 7, 14$

Q.15 (A, B)

$$(P + q)^{99}\ r \leq \frac{99+1}{1+\left|\frac{1/2}{1/2}\right|} \quad \Rightarrow r \leq \frac{100}{2} \quad \Rightarrow \ r \leq 50$$

Terms 50 or 51 are highest

Q.16 (A,C)

(A) $P(A/B) = \frac{P(A\cap B)}{P(B)}$

$$= \frac{P(A)+P(B)-P(A\cup B)}{P(B)} \ge \frac{P(A)+P(B)-1}{P(B)}$$

(B) $P(A\cup B) = P(A)+P(B)-P(A)\,P(B)$

∴ A & B are ind.

$= P(A)\,(1-P(B)+P(B)$

$= P(A)\,P(\overline{B}) + P(B) + 1 - 1$

$= P(A)\,P(\overline{B}) - P(\overline{B}) + 1$

$\Rightarrow\ 1 + P(\overline{B})\,(P(A)-1) = 1 - P(\overline{A})\,P(\overline{B})$

Comprehension # 1 (Q. No. 17 and 18)

Q.17 **(D)**

Probability

$$= \frac{\frac{1}{3}\times\left(\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}\right)}{\frac{1}{3}\times\left[\frac{{}^1C_1\times{}^3C_1}{{}^6C_2}+\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}+\frac{{}^3C_1\times{}^4C_1}{{}^{12}C_2}\right]}$$

$$= \frac{\frac{6}{36}}{\frac{13}{15}+\frac{6}{30}+\frac{12}{66}} = \frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}} = \frac{55}{181}.\ \textbf{Ans.}$$

Q.18 **(A)**

$$B_1\left\langle\begin{array}{l}1\ W\\3\ R\\\underline{2\ B}\\\underline{\ 6\ }\end{array}\right.,\quad B_2\left\langle\begin{array}{l}2\ W\\3\ R\\\underline{4\ B}\\\underline{\ 9\ }\end{array}\right.\ \text{and}\ B_3\left\langle\begin{array}{l}3\ W\\4\ R\\\underline{5\ B}\\\underline{\ 12\ }\end{array}\right.$$

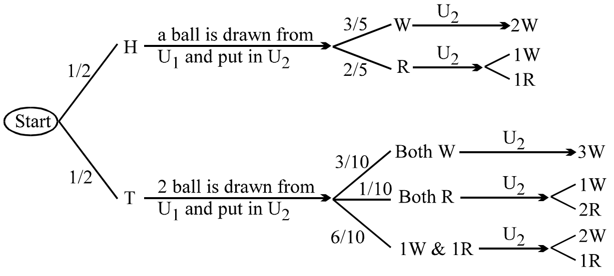
Probability $= \left(\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}\right)+\left(\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}\right)$

$$= \frac{6+36+40}{72\times 9} = \frac{82}{648}.\ \textbf{Ans.}]$$

Comprehension # 2 (Q. No. 19 and 20)

Q.19 (B)

$$U_1\left\langle\begin{array}{l}3W\\2R\end{array}\right. \qquad U_2 \to 1W$$



Required Probability =

$$\left(\frac{1}{2}\times\frac{3}{5}\times 1\right)+\left(\frac{1}{2}\times\frac{2}{5}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{3}{10}\times 1\right)+$$
$$\left(\frac{1}{2}\times\frac{1}{10}\times\frac{1}{3}\right)+\left(\frac{1}{2}\times\frac{6}{10}\times\frac{2}{3}\right)$$

$$= \frac{3}{10}+\frac{1}{10}+\frac{3}{20}+\frac{1}{60}+\frac{12}{60} = \frac{46}{60} = \frac{23}{30}.\ \textbf{Ans.}$$

Q.20 (D)

$$\text{Required Probability} = \frac{\frac{1}{2}\times\frac{3}{5}\times 1+\frac{1}{2}\times\frac{2}{5}\times\frac{1}{2}}{\frac{23}{30}} =$$

$$\frac{4}{10}\times\frac{30}{23} = \frac{12}{23}.\ \textbf{Ans.}$$

Comprehension # 3 (Q. No. 21 to 23)

There are n urns each containing (n + 1) balls such that the i^{th} urn contains i white balls and (n + 1 – i) red balls. Let u_i be the event of selecting i^{th} urn, i = 1, 2, 3, ..., n and w denotes the event of getting a white ball.

Q.21 (B)

Q.22 (A)

Q.23 (B)

(21 to 23) $u_1, u_2, u_3, \ldots\ldots u_n$ are urns.

$$\left.\begin{array}{l}u_1 = 1W + nR\\u_2 = 2W + (n-1)R\\\vdots\\u_i = i(W) + (n+1-i)R\\\vdots\\u_n = nW + 1R\end{array}\right\}\text{urn}$$

21 Given $P(u_i) = k_i$

∴ $P(u_1) = k$, $P(u_2) = 2k$ etc.

∴ $k + 2k + 3k + \ldots\ldots\ldots + nk = 1$

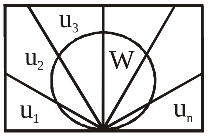
$$k = \frac{1}{\sum n} = \frac{2}{n(n+1)}$$

∴ $P(u_i) = \frac{2i}{n(n+1)}$ (u_i = i^{th} urn is selected)

Now W = white ball is drawn

$$P(W) = \sum_{i=1}^{n} P(u_i \cap W) = \sum_{i=1}^{n} P(u_i).P(W/u_i)$$

$$= \sum_{i=1}^{n} \frac{2i}{n(n+1)}\left(\frac{i}{n+1}\right)$$



=

$$\frac{2}{n(n+1)^2}\sum_{i=1}^{n} i^2 = \frac{2}{n(n+1)^2}.\frac{n(n+1)(2n+1)}{6}$$

$$= \frac{2(2n+1)}{(n+1)6}$$

$$= \lim_{n\to\infty} P(W) = \lim_{n\to\infty} \frac{4n\left(1+\frac{1}{2}\right)}{n\left(1+\frac{1}{n}\right)} = \frac{1}{6} = \frac{2}{3}$$

Ans

22 Given $P(u_i) = C$

⇒ Selecting of any urn in equiprobable

∴ $P(u_1) = P(u_2) = \text{............} = P(u_n) = \frac{1}{n}$

In this case $P(W) = P(u_1 \cap W) + P(u_2 \cap W) + \text{....................} + P(u_n \cap W)$

$= P(u_1) \cdot P(W/u_1) + \text{................}$

$$= \frac{1}{n} \cdot [P(W/u_1) + P(W/u_2) + \text{................}P(W/u_n)]$$

$$= \frac{1}{n}.\left[\frac{1}{n+1} + \frac{2}{n+1} + \text{...........} + \frac{n}{n+1}\right]$$

$$= \frac{n(n+1)}{2n(n+1)} = \frac{1}{2}$$

$$P(u_n/W) = \frac{P(u_n \cap W)}{P(W)} = \frac{P(u_n)\cdot P(W/u_n)}{P(W)}$$

$$= 2\left[\frac{1}{n}.\frac{n}{(n+1)}\right] = \frac{2}{n+1} \text{ Ans.}$$

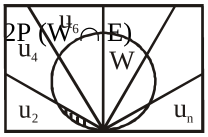
Q.23 Let n = even

E : event that even numbered urn is selected

∴ either an even numbered or an odd numbered urn will be selected and all events are equally likely

∴ $P(E) = \frac{1}{2}$

$$P(W/E) = \frac{P(W \cap E)}{P(E)} = 2P(W \cap E)$$



$P(W/E)$

$$= \frac{1}{n}[P(W/u_2) + P(W/u_4) + \text{..........} + P(W/u_n)]$$

$$= \frac{1}{n}\left[\frac{2}{n+1} + \frac{4}{n+1} + \frac{6}{n+1} + \text{........} + \frac{n}{n+1}\right]$$

$$P(W/E) = \frac{2}{n(n+1)}\left[1 + 2 + 3 + \text{........} + \frac{n}{2}\right]$$

$$= \frac{2}{n(n+1)}\left[\frac{\frac{n}{2}\left(\frac{n}{2}+1\right)}{2}\right] = \frac{n(n+2)}{n(n+1)4}$$

$$P(W/E) = 2.\frac{n+2}{4(n+1)} = \frac{n+2}{2(n+1)} \text{ Ans.}$$

Comprehension # 4 (Q. No. 24 to 26)

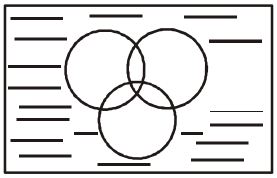
Q.24 (A)

Q.25 (C)

Q.26 (D)

24 $P = 1 - P(A \cup B \cup C)$

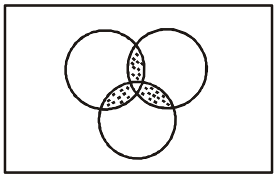
$= 1 - P(A) - P(B) - P(C) + P(A \cap C) + P(C \cap A) - P(A \cap B \cap C)$



$= P(A \cap B \cap C)^1 - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A)$

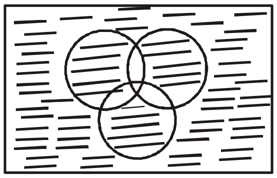
$= P(\overline{A} \cup \overline{B} \cup \overline{C}) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A)$

25



26 $P = 1 - P(A \cap B \cap C)$

$= 1 - P(A \cup B \cup C) + P(A) + P(B) + P(C)$



$- P(A \cup B)$
$= 1 - P(A \cup B \cup C)^c + P(A) + P(B) + P(C) - P(A \cup B)$
$- P(B \text{ È } C) - P(C \cap A)$

If n positive integers taken at random and multiplied together, then the chance that the last digit of the product would be

Q.27 (A) $\to$ (q), (B) $\to$ (r), (C) $\to$ (r), (D) $\to$ (r)

(A) Even integers ends in 0, 2, 4, 6, 8. Square of an even integer ends in 4 only when the integer ends either in 2 or 8.

$\therefore$ probability $= \frac{2}{5}$

(B) $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A).P(B) = \frac{1}{6}$

$P(\overline{A}) = \frac{2}{3}$

$\Rightarrow P(A) = \frac{1}{3}$ $\quad\Rightarrow P(B) = \frac{1}{2}$

$\therefore$ $6P(B/A) = 6P(B) = 3$

(C) Required probability $= p = {}^2C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2}$

$\therefore$ $6p = 3$

(D) $625p^2 - 175p + 12 < 0$ gives $p \in \left(\frac{3}{25}, \frac{4}{25}\right)$

$$\left(\frac{4}{5}\right)^{n-1} . \frac{1}{5} = p$$

$$\therefore \quad \frac{3}{25} < \left(\frac{4}{5}\right)^{n-1} . \frac{1}{5} < \frac{4}{25}$$

i.e. $\frac{3}{5} < \left(\frac{4}{5}\right)^{n-1} < \frac{4}{5}$

value of n is 3

Q.28 (A)-p,q; (B)-r, (C)-p,s

(A) $p = \frac{1}{2}\left(\frac{1}{2}\right)^3 \frac{1}{2} + \left(\frac{1}{2}\right)^6 \frac{1}{2}$

$$= \frac{1/2}{1-(1/2)} = \frac{1/2}{1-1/8}$$

$$= \frac{1/2}{7/8} = \frac{8}{14} = \frac{4}{7}$$

$$q = \frac{1}{2}\frac{1}{2} + \left(\frac{1}{2}\right)^3 \frac{1}{2}\frac{1}{2} + ...$$

$$= \frac{1/2 \times 1/2}{1-(1/2)^3} = \frac{1/4}{7/8} = \frac{8}{28} = \frac{2}{7}$$

$$r = \frac{1/8}{1-1/8} = \frac{1}{7}$$

(B) $p = \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \times \frac{1}{6}$ ....

$$\frac{1}{6}\left[\frac{1}{1-\frac{125}{2.6}}\right] = \frac{1}{6} \times \frac{216}{91} = \frac{36}{91}$$

$$q = \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^7 \times \frac{1}{6} .....$$

$$\frac{5}{6} \times \frac{1}{6}\left[1 + \frac{5^3}{6^3} + \frac{5^6}{6^6} .....\right] = \frac{5}{36} \times \frac{216}{91} = \frac{30}{91}$$

$$r = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} ....$$

$$= \frac{25}{216} \times \frac{216}{91} = \frac{25}{91}$$

(C) $p(A) = \frac{2}{3}, p(B) = \frac{1}{2}, p(C) = \frac{1}{4}$

$$p = \frac{2}{3} + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4}\right) \times \frac{2}{3} + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4}\right)^2 \times \frac{2}{3} .....$$

$$= \frac{2}{3}\left[1 + \frac{3}{24} ....\right] = \frac{2}{3}\left[\frac{1}{1-\frac{3}{24}}\right] = \frac{2}{3} \times \frac{24}{21} = \frac{16}{21}$$

$$q = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} \times \frac{1}{3} \times \frac{1}{2} + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4}\right)^2 \times \frac{1}{6} ....$$

$$= \frac{1}{6}\left[1 + \frac{3}{24} + \frac{3^2}{24^2} ....\right]$$

$$= \frac{1}{6} \times \left(\frac{1}{1-3/24}\right) = \frac{1}{6} \times \frac{24}{21} = \frac{4}{21}$$

$$r = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{2} \times \frac{3}{4}\left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}\right) + \frac{1}{3} \times \frac{1}{2} \times \frac{3}{4}\left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}\right)^2 .....$$

$$= \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}\left[1 + \frac{3}{24} + \frac{3^2}{24^2} ....\right] = \frac{1}{24} \times \frac{24}{21} = \frac{1}{21}$$

Q.29 (A) → (s),(B) → (r),(C) → (q), (D) → (p)

(A) $\mathbf{8}$ **-** $\left.\begin{matrix}6+2\\5+3\\4+4\end{matrix}\right]$ 5 ways ; 3+5 # 1 cong

$$p = \frac{1}{5}$$

(B) A = 2 nd ball in white

B_1 = 1st ball in white

B_2 = 1st is black

$$P\,(B_1 / A) = \frac{p\left(A/B_1\right)p(B_1)}{p(A/B_1)p(B_1)+p(A/B_2)p(B_2)}$$

$$= \frac{\frac{4}{7}\times\frac{3}{6}}{\frac{4}{1}\times\frac{3}{6}+\frac{3}{7}\times\frac{4}{6}}$$

(c) $\frac{2}{5} = (1 – P)\,P + (1 – P)^3\,P + (1\text{-}P)^5\,p + ...$

$$\frac{2}{5} = P\;(1 – P)\{1 + (1 – P)^2 + (1 – P)^4 + ...\}$$

$$= P\;(1\text{-}P)\left(\frac{1-(1-P)^2}{1-(1-P)}\right) = (1\text{-}P - (1\text{-}P)^3 = \frac{2}{5} \text{ solve}$$

for $p = \frac{1}{3}$

(D) (3,3,3,3) or (3,3,3,5) total → 2^4

For = $1+\frac{4!}{3!} = 5$ $p = \frac{5}{2^4}$

NUMERICAL VALUE BASED

Q.1 [10]

$a_1 + a_2 + a_3 + ... + a_7 = 9k,\ k \in I.$ Also $a_1 + a_2 + ... + a_9$

$= 1 + 2 + 5 + ... + 4 = 45$

$\Rightarrow \quad a_8 + a_9 = 45 – 9k \quad \Rightarrow \quad 3 \le a_8 + a_9 \le 17$

$\Rightarrow \quad k = 4 \quad \Rightarrow \quad a_8 + a_9 = 9$

$\Rightarrow \quad (1, 8)\,(2, 7), (3, 6), (4, 5)$

$$P(E) = \frac{4}{36} = \frac{1}{9}.$$

Q.2 [1]

$$P(C) = \frac{1}{{}^4C_1+{}^4C_2+{}^4C_3+{}^4C_4} = \frac{1}{2^4-1} = \frac{1}{15}\ ;$$

P(correct) = 1 – P (all wrong)

$$= 1 - \frac{14}{15}\times\frac{13}{14}\times\frac{12}{13}\times\frac{11}{12}\times\frac{10}{11} = \frac{1}{3}.$$

Q.3 [3]

$P(x) = \frac{2}{3}$ $P(y) = \frac{3}{4}$ P(z) = ? P

; P(2 bultets) = $\frac{11}{24}$

$$\frac{11}{24} = \frac{2}{3}\times\frac{3}{4}\,(1 – P) + \frac{2}{3}\times\frac{1}{4}\times P+\frac{1}{3}\times\frac{3}{4}\times P$$

$$P = \frac{1}{2}$$

Q.4 [58]

$$\left(\frac{13}{52}\right)^2\times\left(\frac{13}{52}\right)^2\times\left(\frac{13}{52}\right)^2\times\frac{6!}{2!2!2!} = P$$

Q.5 [6]

A = Letter drawn is vowel ; B_1 = written by Englishmen ; B_2 = written by American

$$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1)+P(A/B_2)P(B_2)}$$

$$= \frac{0.4\times\frac{3}{6}}{0.4\times\frac{3}{6}+0.6\times\frac{2}{5}} = \frac{5}{11}$$

Q.6 [1]

P(identify high grade tea correctly) = $\frac{9}{10}$; P (identify low grade tea correctly) = $\frac{8}{10}$

P (Given high grade tea) = $\frac{3}{10}$; P (Given low grade tea) = $\frac{7}{10}$

P (Low grade tea / says high grade tea)

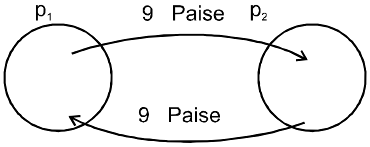
$$= \frac{\frac{7}{10}\times\frac{2}{10}}{\frac{7}{10}\times\frac{2}{10}+\frac{3}{10}\times\frac{9}{10}} = \frac{14}{41}$$

Q.7 [1]

10 coins 9   5 paisa   10 coins 5 paisa
1   1 Rs.

p = p (1 Rs. transfered + Back transfered) + p (1 Rs. not transfered)

$$\frac{{}^{9}c_8 \times {}^{1}c_1}{{}^{10}c_9} \times \frac{{}^{18}c_8 \times {}^{1}c_1}{{}^{19}c_9} + \frac{{}^{9}c_9 \times {}^{19}c_{19}}{{}^{10}c_9 \times {}^{19}c_{19}} = \frac{10}{19}$$



method 2 when 1 Rs coin is in second purse and did not came back in first purse this prob. = $\frac{{}^{9}C_8 \times {}^{l}C_l}{{}^{10}C_9} \times$

$\frac{{}^{18}c_9}{{}^{19}c_9} = \frac{9}{19} \Rightarrow$ Required probability $= 1 - \frac{9}{19} = \frac{10}{19}$

Q.8 [32]

"PARALLELOGRAM" or "PARALLELOPIPED"

$\Rightarrow$ A = RA is visible

B_1 = its from PARALLELOGRAM

$\Rightarrow$ B_2 = its from PARALLELOPIPED

$$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1) + P(A/B_2)P(B_2)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{12}}{\frac{1}{2} \times \frac{2}{12} + \frac{1}{2} \times \frac{1}{13}} = \frac{13}{19}$$

Q.9 [10]

Unit digit of 3^a = 3, 9, 7, 1 each occurs 25 times in (0, 1, 2, ..., 99)

Unit digit of 7^b = 7, 9, 3, 1 each occurs 25 times in (0, 1, 2, ..., 99)

3^a	7^b
1	7

3^a	7^b
7	1

3^a	7^b
9	9

$$\Rightarrow \frac{25}{100} \times \frac{25}{100} + \frac{25}{100} \times \frac{25}{100} + \frac{25}{100} \times \frac{25}{100} = \frac{3}{16}$$

Q.10 [5]

$\frac{7}{12} = \frac{1}{n+1} \times 1 + \frac{n}{n+1} \times \frac{1}{2}$ solve for n we get n = 5

KVPY

PREVIOUS YEAR'S

Q.1 (C)

For A to win, A can draw either 3, 6 or 5,6. If A draws 3, 6 then B can draw only 8 &9

Prob. $= \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

If A draws 5, 6 then B can draw, any two

Probability $= \frac{1}{3} .1 = \frac{1}{3}$

Probability $= \frac{1}{9} + \frac{1}{3} = \frac{4}{9}$

Q.2 (C)

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105 till 98 terms 48 terms are even and 48 terms odd

99th term $= \frac{99 \times 100}{2}$ = even

Total even terms = 48 + 1 = 49

Probability $= \frac{49}{99}$

Q.3 (A)

P1 : 4 copper coins 3 silver coins

P2 : 6 copper coins 4 silver coins

E = Event of copper coin

$P(E) = P(P_1).\ P(E/P_1) + P(P_2).\ P(E/P_2)$

$$= \frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{6}{10} = \frac{41}{70}$$

Q.4 (B)

Ways to make the sum K is coefficient of x^K in $(x + x^2)^{10}$

Coefficient of x^K in $x^{10}(1+ x)^{10}$

Coefficient of x^{K-10} in $(1 +x)^{10}$

Which is ${}^{10}C_{K-10}$

So ways to make sum minimum K is

${}^{10}C_{K-10} + {}^{10}C_{K-9} + {}^{10}C_{K-8} + ...+ {}^{10}C_{10}$

Probability

$$P(K) = \frac{{}^{10}C_{K-10} + {}^{10}C_{K-9} + ... + {}^{10}C_{10}}{2^{10}}$$

$$P(K) = \frac{2^{10} - \left({}^{10}C_0 + ... + {}^{10}C_{K-11}\right)}{2^{10}}$$

$$= 1 - \frac{{}^{10}C_0 + ... + {}^{10}C_{K-11}}{2^{10}} > \frac{1}{2}$$

But K should be maximum so

${}^{10}C_{K-11} = {}^{10}C_5$ (middle value)

So that ${}^{10}C_0 + ... + {}^{10}C_{K-11}$ is max

So K = 16

Q.5 (C)

$$\frac{{}^{100}C_5[1 \times 2(3)]}{{}^{100}C_5 \times 5!}$$

Suppose 1, 2, 3, 4, are selected coupons.

$\boxed{}\boxed{}\boxed{1}\boxed{}\boxed{}$

$= \frac{1}{20}$

place of 1 is fixed

Total arrangements of 5 is 2

and $\boxed{}\boxed{}\boxed{1}\boxed{}\boxed{5}$

arrangements of 2, 3, 4 are

$$\left.\begin{matrix} 2 & 3 & 1 & 4 & 5 \\ 4 & 2 & 1 & 3 & 5 \\ 4 & 3 & 1 & 2 & 5 \end{matrix}\right\} 3\,\text{way}$$

Q.6 (D)

Let x = 1

favourable out comes (1, 1), (1, 2) ....(1, n) no. of favourable out comes when x = 1

$= \left[\frac{n}{1}\right]$

$\therefore$ no. of favorable out comes when x = 1 or y = 1 = 2

$\left[\frac{n}{1}\right] - 1$

$\therefore$ no. of favourable out comes when x = 2 or y = 2 but

$x \neq 1, y \neq 1 = 2\left[\frac{n}{1}\right] - 1$

Similarly

no. of favourable out comes when x = k or y = k but x, $y \not\in \{1, 2, ... k-1\}$

$2\left[\frac{n}{k}\right] - 1$

So probability

$$= \frac{\sum_{k=1}^{n}\left[\frac{n}{k}\right] - (1+1.......n\text{ times})}{n^2} = \frac{2}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right] - \frac{1}{n}$$

Q.7 (B)

$$P = \frac{6}{6}.\frac{5}{6} + \frac{6}{6}.\frac{1}{6}.\frac{1}{6}.\frac{5}{6} + \frac{6}{6}.\frac{1}{6}.\frac{1}{6}.\frac{1}{6}.\frac{1}{6}.\frac{5}{6} + ...\infty$$

$$= \frac{5}{6} + \frac{5}{6^3} + \frac{5}{6^5} + ...$$

$$= \frac{5/6}{1 - \frac{1}{36}} = \frac{30}{35} = \frac{6}{7}$$

Q.8 (D)

Q.9 (D)

$B_1............B_6.$

Required probability

D_1 never Shows'1'

D_2 Shows '1' (one time) Then D_1 shows '1'

$\left(\frac{5}{6}\right)^n$

$\left\{{}^{n}C_1\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right)\right\}\left\{\frac{1}{6}\right\}$

$$\text{Required probability} = \left(\frac{5}{6}\right)^n + n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6^2}\right)$$

Q.10 (C)

$$\frac{\underline{|4}}{\underline{|(2)^2\underline{|2}}} \times \frac{\underline{|6}}{\underline{|(2)^3\underline{|3}}} \times \underline{|6} = 32400$$

Q.11 (C)

Digits are 1, 2, 3, 4, 5

Even digits = 2, 4; number of Even digits =2

Total marks = 2375; number of Odd digits = 3

Sum of any 2 adjacent digits is odd

$\Rightarrow$ Odd and even digits will alternate

Case I For n; Repetition is not allowed

$\Rightarrow$ OEOEO is the only possibility of arrangement of digits, where O = Odd digit, E = Even digit.

So number of Arrangements

$$n = \frac{3}{O} \times \frac{2}{E} \times \frac{2}{O} \times \frac{1}{E} \times \frac{1}{O} = 12$$

Case I For m; Repetition is allowed

$\Rightarrow$ Two possipilities

(a) OEOEO

Number of such arrangements

$$= \frac{3}{O} \times \frac{2}{E} \times \frac{3}{O} \times \frac{2}{E} \times \frac{3}{O} = 108$$

(b) EOEOE

Number of such arrangements

$$= \frac{2}{E} \times \frac{3}{O} \times \frac{2}{E} \times \frac{3}{O} \times \frac{2}{E} = 72$$

So m = 108 + 72 = 180

$$\frac{m}{n} = \frac{180}{12} = 15$$

Q.12 (A)

$$P(b_2) = P(b_1).P\left(\frac{b_2}{b_1}\right) + P(R_1).P\left(\frac{b_2}{R_1}\right)$$

$$= \frac{b}{b+r}.\frac{b+1}{b+r+1} + \frac{r}{b+r}.\frac{b}{b+r+1} = \frac{b}{b+r}$$

Q.13 (D)

3rd time target will hit in sixth time

So, In first 5 attempt these will be 3L, 2W and at 6th attempt shot will be hit

So, $^5C_3\left(\frac{3}{4}\right)^3\times\left(\frac{1}{4}\right)^2\times\left(\frac{1}{4}\right) = \frac{270}{4096} = 0.06591$

Q.14 (A)

$p_1 = 1 -$ (no die show six)

$$1-\left(\left(\frac{5}{6}\right)^6\right) = 0.6651$$

$p_2 = 1 -$ (no die show two + one die shown two)

$$p_2 = 1-\left[\left(\frac{5}{6}\right)^{12} + {}^{12}C_1\left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right)^1\right] = 0.61866$$

$p_1 > p_2$

Q.15 (B)

Case (1) : all tail $\left(\frac{1}{2}\right)^5$

Case (2) : 4T , 1H $^5C_4\left(\frac{1}{2}\right)^4\cdot\left(\frac{1}{2}\right)^5 = \frac{5}{2^5}$

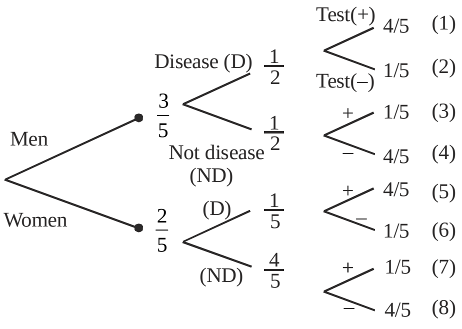
Case (3) :× T × T × T × $\left(\frac{1}{2}\right)^3 \times {}^4C_2\left(\frac{1}{2}\right)^2 = \frac{1}{2^5}\times 6 = \frac{6}{2^5}$

Case (4) : × T × T ×

$$\left(\frac{1}{2}\right)^2\times\left(\frac{1}{2}\right)^3 = \frac{1}{2^5}$$

Overall $\frac{13}{2^5}$

Q.16 (A)



$$\frac{(1)+(3)}{(1)+(3)+(5)+(7)}$$

$$=\frac{\frac{3}{5}\times\frac{1}{2}\times\frac{4}{5}+\frac{3}{5}\times\frac{1}{2}\times\frac{1}{5}}{\frac{3}{5}\times\frac{1}{2}\times\frac{4}{5}\times\frac{3}{5}\times\frac{1}{2}\times\frac{1}{5}+\frac{2}{5}\times\frac{1}{5}\times\frac{4}{5}+\frac{2}{5}\times\frac{4}{5}\times\frac{1}{5}}$$

$$=\frac{\frac{3}{10}}{\frac{3}{10}+\frac{16}{125}} = \frac{75}{107}$$

Q.17 (A)

$A = [\,]_{3\times 3}$

$A^2 = -1$

$|A|^2 = -1$ (Not possible)

A → diagonal matrix

$$\begin{bmatrix} \overset{2001}{\underline{\quad}} & 0 & 0 \\ 0 & \overset{2001}{\underline{\quad}} & 0 \\ 0 & 0 & \overset{2001}{\underline{\quad}} \end{bmatrix}$$

$$P = \frac{(2001)^3}{(2001)^9} = \frac{1}{(2001)^6}$$

$P = (1+2000)^{-6}$

$= \left(2000^{-6}\right)\left(1+\frac{1}{2001}\right)^{-1}$ less then⊥

$= \frac{1}{2^6\times 10^{18}}(\downarrow)$

$P < \frac{1}{10^{18}}$

Q.18 (A)

Case-I : RRBB, RRBB
RRRBB, RBB
RRRB, RBBB
RRRRB, BBB
RRRR, BBBB

$$=\frac{1}{2}\times\frac{2}{5}\times\frac{1}{4}\times\frac{1}{5} = \frac{1}{100}$$

Case-II $= \frac{1}{100}$ similarly

$p = \frac{1}{100} + \frac{1}{100} = 2\%$

Q.19 **(A)**

For equal roots $b^2 = 16c \Rightarrow b = 4\sqrt{c}$ (as $b > 0$).

Hence c should be a perfect square

so probability $= \frac{10}{100\times 100} = 0.001$

Q.20 **(C)**

All even digit numbers in B = {2, 4, 6, 8} fav : forming a 4 digit nos with all digit is even and not divisible by 4

2222, 6666, 2266,

$\frac{4!}{2!2!} = 6$ cases

(2226), (2666)

$\frac{4!}{3!} = 4$ cases $\quad \frac{4!}{3!} = 4$ cases

Total = 1 + 1 + 6 + 4 + 4 = 16 cases

Total : forming a 4-digit no. from {1, 2, ...,9}

but not divisible by 4

C-1 all 4 digit are add = $5^4 = 625$

C-2 all 4 digit are even = 16

C-3 one even & 3 odd

you can not take 2 or 6 as one of even digit (12, 16, 32, 36, ...) all are divisible by 4

But you can take 4 or 8 as one of even digit ....

${}^2C_1 \times {}^4C_1 \times 5^3 = 1000$

Take one even digit out of 4 & 8

select one place for 4 & 8 out of 4-places

Total = 1000 + 625 + 16 = 1641

$$P = \frac{\text{fav.}}{\text{total}} = \frac{16}{1641}$$

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (1)

a, b, c Î ∈ {1,2,3,4,5,6}

n(s) = 6 × 6 × 6 = 216

$D = 0 \Rightarrow b^2 = 4ac$

$ac = \frac{b^2}{4}$ $\quad$ If b = 2, ac = 1 ⇒ a = 1, c = 1

If b = 4, ac = 4 ⇒ a = 1, c = 4

a = 4, c = 1

a = 2, c = 2

If b = 6, ac = 9 ⇒ a = 3, c = 3

$\therefore$ probability = $\frac{5}{216}$

Q.2 [0.125]

$$\text{Prob.} = \left(\frac{2}{3} \cdot \frac{3}{4}\right)^3 = \frac{1}{8}$$

Q.3 (1)

$$n(A) = 7......... + \underbrace{.................}_{7}$$

$= 1 \times 9 \times 9 \times 9 + 8 \times {}^3C_1 \times 1 \times 9 \times 9$

$= 729 + 1944 = 2673$

$$\text{Favourable : } ....7 + \underbrace{................2}_{7\ \text{exavtly once}}$$

$= 8 \times 9 \times 9 + 1 \times 9 \times 9 \times 1 + 2 \times 8 \times 1 \times 9$

$= 648 + 81 + 144$

$= 873$

$$\therefore \quad \text{Probability} = \frac{873}{2973} = \frac{97}{297}$$

Q.4 [0.5957]

$$\text{Nonveg + smoker } \frac{160}{400} \xrightarrow{\text{disease}} \frac{160}{400} \times \frac{35}{100}$$

$$\text{Veg + smoker } \frac{140}{400} \xrightarrow{\text{disease}} \frac{140}{400} \times \frac{20}{100}$$

$$\text{Veg + smoker } \frac{100}{400} \xrightarrow{\text{disease}} \frac{100}{400} \times \frac{10}{100}$$

Required probability =

$$\frac{\frac{160 \times 35}{400 \times 100}}{\frac{160 \times 35}{400 \times 100} + \frac{140 \times 20}{400 \times 100} + \frac{100 \times 10}{400 \times 100}}$$

$$= \frac{16 \times 35}{16 \times 35 + 14 \times 20 + 100} = \frac{560}{940} = \frac{56}{94} = \frac{28}{47}$$

= 0.5957

Q.5 (4)

Required probability

$$= \frac{{}^5C_2 \times 3^3}{4^5}$$

$$= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$$

Q.6 (1)

$${}^nC_9 \times \left(\frac{1}{2}\right)^9 \times \left(\frac{1}{2}\right)^{n-9} = {}^nC_7 \times \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^{n-7}$$

${}^nC_9 = {}^nC_7 \Rightarrow n = 16$

$$P(\text{2Heads}) = {}^{16}C_2 \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{14}$$

$$= {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$$

Q.7 (2)

$$n(S)=\frac{7!}{2!3!2!}$$

$$n(E)=\frac{6!}{2!2!2!}$$

$$P(E)=\frac{n(E)}{n(S)}=\frac{6!}{7!}\times\frac{2!3!2!}{2!2!2!}$$

$$=\frac{1}{7}\times 3=\frac{3}{7}$$

Q.8 [6]

Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1-y)(1-z)=\alpha \Rightarrow \quad y(1-x)(1-z)=\beta$

$\Rightarrow z(1-x)(1-y)=\gamma \Rightarrow \quad (1-x)(1-y)(1-z)=P$

Putting in the given relation we get x = 2y and y = 3z

$$\Rightarrow \quad x=6z \Rightarrow x=6z \Rightarrow \frac{x}{2}=6$$

Q.9 (3)

P(odd no. twice) = P(even no. thrice)

$$\Rightarrow {}^{n}C_2\left(\frac{1}{2}\right)^n={}^{n}C_3\left(\frac{1}{2}\right)^n \Rightarrow n=5$$

success is getting an odd number then P(odd successes) = P(1) + P(3) + P(5)

$$={}^{5}C_1\left(\frac{1}{2}\right)^5+{}^{5}C_3\left(\frac{1}{2}\right)^5+{}^{5}C_5\left(\frac{1}{2}\right)^5$$

$$=\frac{16}{2^5}=\frac{1}{2}$$

Q.10 (3)

E_1 :Event denotes spade is missing

$$P(E_1)=\frac{1}{4};\ P\left(\overline{E}_1\right)=\frac{3}{4}$$

A : Event drawn two cards are spade

$$P(A)=\frac{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right)+\frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)+\frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right)+\frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}$$

$$=\frac{39}{50}$$

Q.11 (2)

Total cases :

$6\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2$

$n(s)=6\cdot 6!$

Favourable cases :

Number divisible by 3 ≡

Sum of digits must be divisible by 3

Case-I

1, 2, 3, 4, 5, 6

Number of ways = 6!

Case-II

0, 1, 2, 4, 5, 6

Number of ways = 5·5!

Case-III

0, 1, 2, 3, 4, 5

Number of ways = 5·5!

n(favourable) = 6! + 2·5·5!

$$P\ \frac{6!+2\cdot 5\cdot 5!}{6\cdot 6!}=\frac{4}{9}$$

Q.12 (2)

n(E) = 5 + 4 + 4 + 3 + 1 = 17

So, P (E) = $\frac{17}{36}$

Q.13 [6]

Let $P(E_1)=P_1$; $P(E_2)=P_2$; $P(E_3)=P_3$

$P(E_1\cap\overline{E}_2\cap\overline{E}_3)=\alpha=P_1(1-P_2)(1-P_3)$ ......(1)

$P(\overline{E}_1\cap E_2\cap\overline{E}_3)=\beta=(1-P_1)P_2(1-P_3)$ ......(2)

$P(\overline{E}_1\cap\overline{E}_2\cap E_3)=\gamma=(1-P_1)(1-P_2)P_3$ ......(3)

$P(\overline{E}_1\cap\overline{E}_2\cap\overline{E}_3)=P=(1-P_1)(1-P_2)(1-P_3)$ ......(4)

Given that, $(\alpha-2\beta)P=\alpha\beta$

$\Rightarrow (P_1(1-P_2)(1-P_3)-2(1-P_1)P_2(1-P_3))$

$P=P_1P_2$

$(1-P_1)(1-P_2)(1-P_3)_2$

$\Rightarrow (P_1(1-P_2)-2(1-P_1)P_2)=P_1P_2$

$\Rightarrow (P_1-P_1P_2-2P_2+2P_1P_2)=P_1P_2$

$\Rightarrow P_1=2P_2$ ......(1)

and similarly, $(\beta-3\gamma)P=2B\gamma$

$P_2=3P_3$ ......(2)

So, $P_1=6P_3 \Rightarrow \boxed{\frac{P_1}{P_3}=6}$

Q.14 (4)

$$\underset{\text{odd place}}{1}\quad\underset{\text{even place}}{0}\quad\underset{\text{odd place}}{0}\quad\underset{\text{even place}}{1}$$

$$\text{or}\quad\underset{\text{even place}}{1}\quad\underset{\text{odd place}}{0}\quad\underset{\text{even place}}{0}\quad\underset{\text{odd place}}{1}$$

$$\Rightarrow \left(\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}\right)+\left(\frac{2}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\right)$$

$$\Rightarrow \frac{1}{9}$$

Q.15 (1)

$P(X = 1) = {}^5C_1.p.q^44 = 0.4096$

$P(X = 2) = {}^5C_2.p^2.q^3 = 0.2048$

$\Rightarrow \frac{q}{2p} = 2$

$\Rightarrow q = 4p$ and $p + q = 1$

$\Rightarrow p = \frac{1}{5}$ and $q = \frac{4}{5}$

Now

$$P(X = 3) = {}^5C_3.\left(\frac{1}{5}\right)^3.\left(\frac{4}{5}\right)^2 = \frac{10\times16}{125\times25} = \frac{32}{625}$$

Q.16 (1)

Q.17 (2)

Q.18 [4]

Q.19 (3)

Q.20 (1)

Q.21 (2)

Q.22 (2)

Q.23 (2)

Q.24 (2)

Total ways of choosing square = ${}^{64}C_2$

$$= \frac{64\times63}{2\times1} = 32\times63$$

ways of choosing two squares having common side $= 2\,(7\times8) = 112$

Required probability $= \frac{112}{32\times63} = \frac{16}{32\times9} = \frac{1}{18}$.

Ans. (2)

Q.25 [781]

Q.26 (1)

Q.27 [28]

Q.28 (4)

Q.29 (4)

Q.30 (3)

Q.31 (2)

Q.32 [61]

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PREVIOUS YEAR'S

Comprehension # 1 (Q. No. 1 & 2)

Q.1 (B)

$P(\text{white}) = P\,(H\cap \text{white}) + P(T\cap\ \text{white})$

$$\frac{1}{2}\cdot\left\{\frac{3}{5}\times1+\frac{2}{5}\cdot\frac{1}{2}\right\}+\frac{1}{2}\cdot\left\{\frac{{}^3C_2}{{}^5C_2}\times1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}+\frac{{}^3C_1\cdot{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right\}$$

$$= \frac{1}{2}\times\frac{8}{10}+\frac{1}{2}\times\left\{\frac{3}{10}+\frac{1}{30}+\frac{12}{30}\right\} = \frac{4}{10}+\frac{1}{2}\times\frac{22}{30} = \frac{23}{30}$$

Q.2 (D)

$$P\left(\text{Head}/\text{White}\right) = \frac{P(\text{Head}\cap\text{white})}{P(\text{white})}$$

$$= \frac{\frac{1}{2}\times\left\{\frac{3}{5}\times1+\frac{2}{5}\times\frac{1}{2}\right\}}{\frac{23}{30}} = \frac{\frac{4}{10}}{\frac{23}{30}} = \frac{12}{23}$$

Q.3 (A, D)

$P(E\cap F) = P(E)\,.\,P(F)$ ....(1)

$P(E\cap \overline{F}\,) + P(\overline{E}\ \cap F) = \frac{11}{25}$ ....(2)

$P(\overline{E}\cap\overline{F}) = \frac{2}{25}$ ....(3)

by (2)

$P(F) + P\,(E) - 2P\,(E\cap F) = \frac{11}{25}$ ....(4)

by (3)

$1 - [P(E) + P\,(F) - P\,(E\cap F)] = \frac{2}{25}$

$[P(E) + P\,(F) - P\,(E\cap F)] = \frac{23}{25}$ ....(5)

by (4) & (5) $P\,(E)\,P\,(F) = \frac{12}{25}$ ....(6)

and $P\,(E) + P\,(F) = \frac{7}{5}$ ....(7)

By (6) and (7) $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{5}$

or $P(E) = \frac{3}{5}$, $P(F) = \frac{4}{5}$

Q.4 (BD)

$P(x_1) = \frac{1}{2}$ **;** $P(x_2) = \frac{1}{4}$; $P(x_3) = \frac{1}{4}$

$P(x) = P(E_1\,E_2\,E_3) + P(\overline{E}_1\,E_2\,E_3) + P(E_1\,\overline{E}_2\,E_3) +$

$P(E_1\,E_2\,\overline{E_3}) =$

$$\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{3}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{3}{4}$$

$P(x) = \frac{1}{4} \Rightarrow$ (A) $P\left(\frac{x_1^{\,c}}{x}\right) = \frac{P(x_1^{\,c} \cap x)}{P(x)} =$

$$\frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$$

(B) $P(\text{exactly two} / x) = \frac{P(\text{exactly two} \cap x)}{P(x)} =$

$$\frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{3}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{4}} = \frac{7}{8}$$

(C) $P(x/x_2) = \frac{P(x \cap x_2)}{P(x_2)}$

$$= \frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{4}} = \frac{5}{8}$$

(D) $P(x/x_1) = \frac{P(x \cap x_1)}{P(x_1)} =$

$$\frac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{3}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{2}} = \frac{7}{16}$$

Q.5 (A)

Favourable : D_4 shows a number and

only 1 of $D_1D_2D_3$ shows same number

or only 2 of $D_1D_2D_3$ shows same number

or all 3 of $D_1D_2D_3$ shows same number

Required Probability =

$$\frac{{}^6C_1({}^3C_1 \times 5 \times 5 + {}^3C_2 \times 5 + {}^3C_3)}{216 \times 6} =$$

$$\frac{6 \times (75+15+1)}{216 \times 6} = \frac{6 \times 91}{216 \times 6} = \frac{91}{216}$$

Q.6 (AB)

$P(X/Y) = \frac{1}{2}$

$\Rightarrow \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$

$\Rightarrow P(Y) = \frac{1}{3}$

$\Rightarrow P(Y/X) = \frac{1}{3}$

$\Rightarrow \frac{P(X \cap Y)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{2}{3}$

A is correct

$P(X \cap Y) = P(X) \cdot P(Y) \Rightarrow$ X and Y are independent

B is correct

$P(X^c \cap Y) = P(Y) - P(X \cap Y)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

D is not correct

Q.7 (A)

P (problem solved by at least one) = 1 – P(problem is not solved by by all)

$= 1 - P(\overline{A})\ P(\overline{B})\ P(\overline{C})\ P(\overline{D}) = 1 -$

$$\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{7}{8}\right) = 1 - \frac{21}{256} = \frac{235}{256}$$

Q.8 [6]

Let x, y, z be probability of E_1, E_2, E_3 respectively

$\Rightarrow x(1-y)(1-z) = \alpha$ $\quad\Rightarrow y(1-x)(1-z) = \beta$

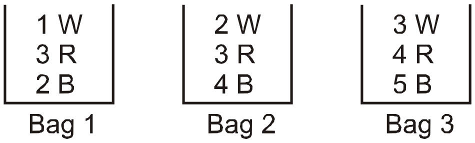
$\Rightarrow z(1-x)(1-y) = \gamma$ $\quad\Rightarrow (1-x)(1-y)(1-z) = P$

Putting in the given relation we get x = 2y and y = 3z

$\Rightarrow x = 6z$ $\quad\Rightarrow \frac{x}{z} = 6$

Comprehension # 2 (Q. No. 9 & 10)

Q.9 (A)



$\Rightarrow P(W\,W\,W) + P(R\,R\,R) + P(B\,B\,B)$

$$\left(\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}\right)+\left(\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}\right)+\left(\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}\right)$$

$\Rightarrow \frac{6+36+40}{6 \times 9 \times 12}$ $\quad\Rightarrow \frac{82}{648}$

Q.10 (D)

P (Ball drawn from box 2 / one is W one is R)

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3} \times \frac{2 \times 3}{{}^{9}C_{2}}}{\frac{1}{3}\left[\frac{1 \times 3}{{}^{6}C_{2}} + \frac{2 \times 3}{{}^{9}C_{2}} + \frac{3 \times 4}{{}^{12}C_{2}}\right]}$$

$$= \frac{\frac{2 \times 3 \times 2}{9 \times 8}}{\frac{3 \times 2}{6 \times 5} + \frac{2 \times 6 \times 2}{9 \times 8} + \frac{3 \times 4 \times 2}{12 \times 11}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{5} + \frac{1}{6} + \frac{2}{11}} = \frac{\frac{1}{6}}{\frac{66 + 55 + 60}{55 \times 60}} = \frac{55}{181}$$

Q.11 (A)

3 Boys & 2 Girls..................

(1) B (2) B (3) B (4)

Girl can't occupy 4th position. Either girls can occupy 2 of 1, 2, 3 position or they can both be a position (1) or (2).

Hence total number of ways in which girls can be seated is ${}^{3}C_{2}$ × 2! × 3! + ${}^{2}C_{1}$ × 2! × 3! = 36 + 24 = 60.

Number of ways in which 3 B & 2 A can be seated = 5 !

Hence required prob. = $\frac{60}{5!} = \frac{1}{2}$.

Comprehension # 3 (Q. No. 12 & 13)

Q.12 (B)

$x_1 + x_2 + x_3$ is odd if all three are odd or 2 are even & one is odd

$$\frac{2}{3} \times \frac{3}{5} \times \frac{4}{7} + \frac{2}{3} \times \frac{2}{5} \times \frac{3}{7} + \frac{1}{3} \times \frac{3}{5} \times \frac{3}{7} + \frac{1}{3} \times \frac{2}{5} \times \frac{4}{7}$$

$$= \frac{24 + 12 + 9 + 8}{105} = \frac{53}{105}$$

Q.13 (C)

$2x_2 = x_1 + x_3$.

If x_1 & x_3 both are odd 2 × 4 = 8 ways

x_1 & x_3 both are even 1 × 3 = 3 ways

Total = 11 ways

Total $(x_1\ x_2\ x_3)$ triplets are 3 × 5 × 7 $\Rightarrow$ P = $\frac{11}{105}$

Q.14 **[8]**

Let coin is tossed n times

P(atleast two heads) = 1 –

$$\left(\frac{1}{2}\right)^{n} - {}^{n}C_{2} \cdot \left(\frac{1}{2}\right)^{n} \geq 0.96$$

$$\Rightarrow \frac{4}{100} \geq \frac{n+1}{2^{n}}$$

$$\Rightarrow \frac{n+1}{2^{n}} \leq \frac{1}{25}$$

$$\Rightarrow \frac{2^{n}}{n+1} \geq 25$$

$\Rightarrow$ least value of n is 8.

Comprehension # 4 (Q. No. 15 & 16)

Q.15 (A,B)

$$\text{Box} - \text{I} < \begin{matrix} \text{Red} \to n_1 \\ \text{Black} \to n_2 \end{matrix}$$

$$\text{Box} - \text{II} < \begin{matrix} \text{Red} \to n_3 \\ \text{Black} \to n_4 \end{matrix}$$

$$P(R) = \frac{1}{2} \cdot \frac{n_1}{n_1 + n_2} + \frac{1}{2} \cdot \frac{n_3}{n_3 + n_4}$$

$$R(II/R) = \frac{\frac{1}{2} \cdot \frac{n_3}{n_3 + n_4}}{\frac{1}{2} \cdot \frac{n_1}{n_1 + n_2} + \frac{1}{2} \cdot \frac{n_3}{n_3 + n_4}}$$

$$= \frac{\frac{n_3}{n_3 + n_4}}{\frac{n_1}{n_1 + n_2} + \frac{n_3}{n_3 + n_4}}$$

by option $n_1 = 3$, $n_2 = 3$, $n_3 = 5$, $n_4 = 15$

$$P(II/R) = \frac{\frac{5}{20}}{\frac{3}{6} + \frac{5}{20}} = \frac{n_4}{\frac{1}{2} + \frac{1}{4}} = \frac{1}{4} \times \frac{4}{2+1} = \frac{1}{3}$$

Q.16 (C, D)

Given

$$\frac{n_1}{n_1 + n_2} \cdot \frac{n_1 - 1}{n_1 + n_2 - 1} + \frac{n_2}{n_1 + n_2} \cdot \frac{n_1}{n_1 + n_2 - 1} = \frac{1}{3}$$

$3(n_1^2 - n_1 + n_1 n_2) = (n_1 + n_2)(n_1 + n_2 - 1)$

$3n_1(n_1 + n_2 - 1) = n_1 + n_2(n_1 + n_2 - 1)$

$2n_1 = n_2$

Q.17 (C)

$$P(T_1)=\frac{20}{100}, \qquad P(T_2)=\frac{80}{100}$$

$$P(D)=P(T_1)P(D/T_1)+P(T_2)P(D/T_2)$$

$$\Rightarrow \frac{7}{100}=\frac{20}{100}x+\frac{80}{100}y$$

$$\Rightarrow \qquad 20x+80y=7$$

Also $x = 10y$

$$\Rightarrow y=\frac{1}{40} \ \& \ x=\frac{1}{4}$$

Hence, Require probability = $\dfrac{80\times 39}{20\times 30+80\times 39}=\dfrac{78}{93}$

Comprehension # 5 (Q. No. 18 & 19)

Q.18 (B)

$P(X>Y)=P(WW, WD, DW)=P(WW)+P(WD)+P(DW)$

$$=\frac{1}{2}\times\frac{1}{2}+\frac{1}{6}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{6}=\frac{5}{12}$$

Q.19 (C)

$P(X = Y) = P(WL) + P(DD) + P(LW) =$

$$\frac{1}{2}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}=\frac{13}{36}$$

Q.20 (A,B)

$$\frac{P(X\cap Y)}{P(Y)}=\frac{1}{2}$$

$$\frac{P(X\cap Y)}{P(X)}=\frac{2}{5}$$

$$P(X\cap Y)\frac{P(Y)}{2}=\frac{2}{5}P(X)=\frac{2}{5}.\frac{1}{3}=\frac{2}{15}$$

$$\Rightarrow P(Y)=\frac{4}{15}$$

$$\frac{P(\overline{X}\cap Y)}{P(Y)}\frac{P(Y)-P(X\cap Y)}{P(Y)}=\frac{\frac{4}{15}-\frac{2}{15}}{\frac{4}{15}}=\frac{2}{4}=\frac{1}{2}$$

$$P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)$$

$$=\frac{1}{3}+\frac{4}{15}-\frac{2}{15}=\frac{7}{15}$$

Q.21 **(C)**

$x + y + z = 10$

Total number of non-negative solutions = ${}^{10+3-1}C_{3-1}$ $= {}^{12}C_2 = 66$

Now Let $z = 2n$.

$x+y+2n=10\,;\, n\geq 0$

Total number of non-negative solutions = 11 + 9 + 7 + 5 + 3 +1 = 36

Required probability $=\dfrac{36}{66}=\dfrac{6}{11}$

Comprehension # 6 (Q. No. 22 & 23)

Q.22 (A)

Required probability $=\dfrac{4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)}{5!}=\dfrac{9}{120}=\dfrac{3}{40}$

Q.23 (C)

$$n(T_1\cap T_2\cap T_3\cap T_4)=\text{Total}-\ n(\overline{T}_1\cup\overline{T}_2\cup\overline{T}_3\cup\overline{T}_4)$$

$$=5!-\left({}^4C_1 4!2!-\left({}^3C_1.3!2!+{}^3C_1 3!2!2!\right)+\left({}^2C_1 2!2!+{}^4C_1.2.2!\right)-2\right)$$

$=14$

Probability $=\dfrac{14}{5!}=\dfrac{7}{60}$

Q.24 [422.00]

$$P\left(\frac{B}{A}\right)=P(B)$$

$$\Rightarrow \frac{n(A\cap B)}{n(A)}=\frac{n(B)}{n(s)}$$

.........(1)

$\Rightarrow$ n(A) should have 2 or 3 as prime factors

$\Rightarrow$ n(A) can be 2, 3, 4 or 6 as n (A) > 1

n(A) = 2does not satisfy the constraint (1).

for n(A) = 3. n(B) = 2 and n $A\cap B$ = 1

$\Rightarrow$ No. of ordered pair = ${}^6C_4\times\dfrac{4!}{2!}$ = 180

for n(A) = 4. n(B) = 3 and n $(A\cap B)$ = 2

$\Rightarrow$ No. of ordered pairs = ${}^6C_5\times\dfrac{5!}{2!2!}$ = 180

for n(A) = 6. n(B) can be 1,2,3,4 5.

$\Rightarrow$ No. of ordered pairs = $2^6 - 2 = 62$

Total ordered pair = 180 + 180 + 62 = 422.

Q.25 [0.50]

$n(E_2) = {}^9C_2$ (as exactly two cyphers are there)

Now, det A = 0, when two cyphers are in the same column or same row

$\Rightarrow n(E_1 \cap E_2) = 6 \times {}^3C_2.$

$$\text{Hence, } P\left(\frac{E_1}{E_2}\right) = \frac{n(E_1 \cap E_2)}{n(E_2)} = \frac{18}{36} = \frac{1}{2} = 0.50$$

Q.26 (B,C)

Ball	Balls composition	P(B$_i$)
B_1	5R + 5G	$\frac{3}{10}$
B_2	3R + 5G	$\frac{3}{10}$
B_3	5R + 3G	$\frac{4}{10}$

$$(1)\ P(B_3 \cap G) = P\left(\frac{G_1}{B_3}\right)P(B_3) = \frac{3}{8} \times \frac{4}{10} = \frac{3}{20}$$

$$(2)\ P(G) = P\left(\frac{G_1}{B_1}\right)P(B_1) + P\left(\frac{G}{B_2}\right)P(B_2) + P\left(\frac{G}{B_3}\right)P(B_3)$$

$$= \frac{3}{20} + \frac{3}{16} + \frac{3}{20} = \frac{39}{80}$$

$$(3)\ P\left(\frac{G}{B_3}\right) = \frac{3}{8}$$

$$(4)\ P\left(\frac{B_3}{G}\right) = \frac{P(G \cap B_3)}{P(G)} = \frac{3/20}{39/80} = \frac{4}{13}$$

Q.27 (B)

$$P(H) = \frac{2}{3} \text{ for } C_1$$

$$P(H) = \frac{1}{3} \text{ for } C_2$$

For C_1

No. of Heads (α)	0	1	2
Probability	1/9	4/9	4/9

for C_2

No. of Heads (β)	0	1	2
Probability	4/9	4/9	1/9

for real and equal roots

$\alpha^2 = 4\beta$

$(\alpha, \beta) = (0, 0), (2, 1)$

$$\text{So, probability } = \frac{1}{9} \times \frac{1}{4} + \frac{4}{9} \times \frac{4}{9} = \frac{20}{81}$$

Q.28 [6]

$$\text{Let P(r) = probability of r successes } = {}^nC_r\left(\frac{3}{4}\right)^r\left(\frac{1}{4}\right)^{n-r}$$

1–(P(0)+P(1)+P(2)) ≥ 0.95

$$\Rightarrow 1 - {}^nC_0\left(\frac{1}{4}\right)^n - {}^nC_1\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{n-1} - {}^nC_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^{n-2} \geq 0.95$$

$$\Rightarrow 1 - \left(\frac{1 + 3n + \frac{9n(n-1)}{2}}{4^n}\right) \geq 0.95$$

$$\Rightarrow 9n^2 - 3n + 2 \leq 0.05 \times 4^n \times 2 \leq \frac{4^n}{10}$$

for	n=5	212≤102.4	(Not true)
for	n=6	308≤409.6	true

∴ least value of n=6

Q.29 [8.00]

Prime : 2, 3, 5, 7, 11

1 2 4 6 2

$$P(\text{Prime}) = \frac{15}{36}$$

Perfect square 4, 9 $\quad$ $P(\text{perfect square}) = \frac{7}{36}$

3 4

required probability

$$= \frac{\frac{4}{36} + \frac{14}{36} \times \frac{4}{36} + \left(\frac{14}{36}\right)^2 \frac{4}{36} + \ldots.}{\frac{7}{36} + \frac{14}{36} \times \frac{7}{36} + \left(\frac{14}{36}\right)^2 \frac{7}{36} + \ldots.}$$

$$P = \frac{4}{7}$$

$$\therefore\ 14P = 14.\frac{4}{7} = 8$$

Q.30 (A)

$$P = \frac{p(S_1 \cap (E_1 = E_3))}{P(E_1 = E_3)} = \frac{P(B_{1,2})}{P(B)}$$

$$P(B) = \underset{\substack{\uparrow \\ \text{If 1,2} \\ \text{chosen} \\ \text{at start}}}{P(B_{1,2})} + \underset{\substack{\uparrow \\ \text{If 1,3} \\ \text{chosen} \\ \text{at start}}}{P(B_{1,3})} + \underset{\substack{\uparrow \\ \text{If 2,3} \\ \text{chosen} \\ \text{at start}}}{P(B_{2,3})}$$

$$P(B_{1,2}) = \frac{1}{3} \times \underbrace{\frac{1 \times {}^3C_1}{{}^4C_2}}_{\substack{\text{1 is definitely} \\ \text{chosen from } F_2}} \times \underbrace{\frac{1}{{}^5C_2}}_{\substack{\text{1,2 chosen} \\ \text{from } G_2}}$$

$$P(B_{1,3}) = \frac{1}{3} \times \underbrace{\frac{1 \times {}^2C_1}{{}^3C_2}}_{\substack{\text{1 is definitely} \\ \text{chosen from } F_2}} \times \underbrace{\frac{1}{{}^5C_2}}_{\substack{\text{1,2 chosen} \\ \text{from } G_2}}$$

$$P(B_{2,3}) = \frac{1}{3} \times \left[\underbrace{\frac{{}^3C_2 \times 1}{{}^4C_2}}_{\substack{\text{1 is not chosem} \\ \text{from } F_2}} \times \frac{1}{{}^4C_2} + \underbrace{\frac{1 \times {}^3C_1}{{}^4C_2} \times \frac{1}{{}^5C_2}}_{\substack{\text{If 1 is chosen} \\ \text{from } F_2}} \right]$$

$$\frac{P(B_{1,2})}{P(B)} = \frac{1}{5}$$

Q.31 [76.25]

p_1 = probability that maximum of chosen numbers is at least 81

p_1 = 1 – probability that maximum of chosen number is at most 80

$$p_1 = 1 - \frac{80 \times 80 \times 80}{100 \times 100 \times 100} = 1 - \frac{64}{125}$$

$$p_1 = \frac{61}{125}$$

$$\frac{625p_1}{4} = \frac{625}{4} \times \frac{61}{125} = \frac{305}{4} = 76.25$$

the value of $\frac{625p_1}{4}$ is 76.25

Q.32 [24.50]

p_2 = probability that minimum of chosen numbers is at most 40

= 1 – probability that minimum of chosen number is at lest 41

$$= 1 - \left(\frac{600}{100}\right)^3$$

$$= 1 - \frac{27}{125} = \frac{98}{125}$$

$$\therefore \frac{125}{4} p_2 = \frac{125}{4} \times \frac{98}{125} = 24.50$$

Q.33 [214]

A=set of numbers divisible by 3

A={3, 6, 9, 12, ...1998}

$\therefore$ n(A)=666

B=set of numbers divisible by 7

B={7, 14, 21, ... 1995}

$\therefore$ n(B)=285

$A \cap B = \{21, 42, ... 1995\}$

$\therefore n(A \cup B) = 606 + 285 - 95 = 856$

required probability $= \frac{856}{2000} = P$

so, 500 $P = \frac{856}{2000} \times 500 = 214$

Q.34 (A,B,C)

$P(E) = \frac{1}{8}$; $P(F) = \frac{1}{6}$; $P(G) = \frac{1}{4}$; $P(E \cap F \cap G) = \frac{1}{10}$

(C) $P(E \cup F \cup G)$

$= P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(G \cap E) + P(E \cap F \cap G)$

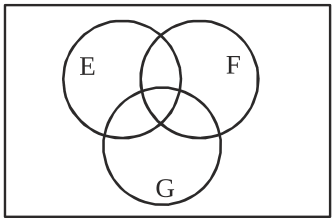
$$= \frac{1}{8} + \frac{1}{6} + \frac{1}{4} - \sum P(E \cap F) + \frac{1}{10}$$

$$= \frac{3+4+6}{24} + \frac{1}{10} - \sum P(E \cap F) = \frac{13}{24} + \frac{1}{10} - \sum P(E \cap F)$$

$\Rightarrow P(E \cup F \cup G) \le \frac{13}{24}$ [(C) is Correct]

(D) $P(E^c \cap F^c \cap G^c) = 1 - P(E \cup F \cup G) \ge 1 - \frac{13}{24}$

$\Rightarrow P(E^c \cap F^c \cap G^c) \ge \frac{11}{24}$ [(D) is Incorrect]



(A) $P(E) = \frac{1}{8} \ge P(E \cap F \cap G^C) + P(E \cap F \cap G)$

$\Rightarrow \frac{1}{8} \ge P(E \cap F \cap G^C) + \frac{1}{10} \Rightarrow \frac{1}{8} - \frac{1}{10} \ge P(E \cap F \cap G^c)$

$\Rightarrow \frac{1}{40} \ge P(E \cap F \cap G^C)$ [(A) is correct]

(B) $P(E) = \frac{1}{6} \ge P(E^C \cap F \cap G) + P(E \cap F \cap G)$

$\Rightarrow \frac{1}{6} - \frac{1}{10}\ P(E^C \cap F \cap G)$

$\Rightarrow \frac{4}{60} \ge P(E^C \cap F \cap G)$

$\Rightarrow \frac{1}{15} \ge P(E^C \cap F \cap G)$ [(B) is Correct]